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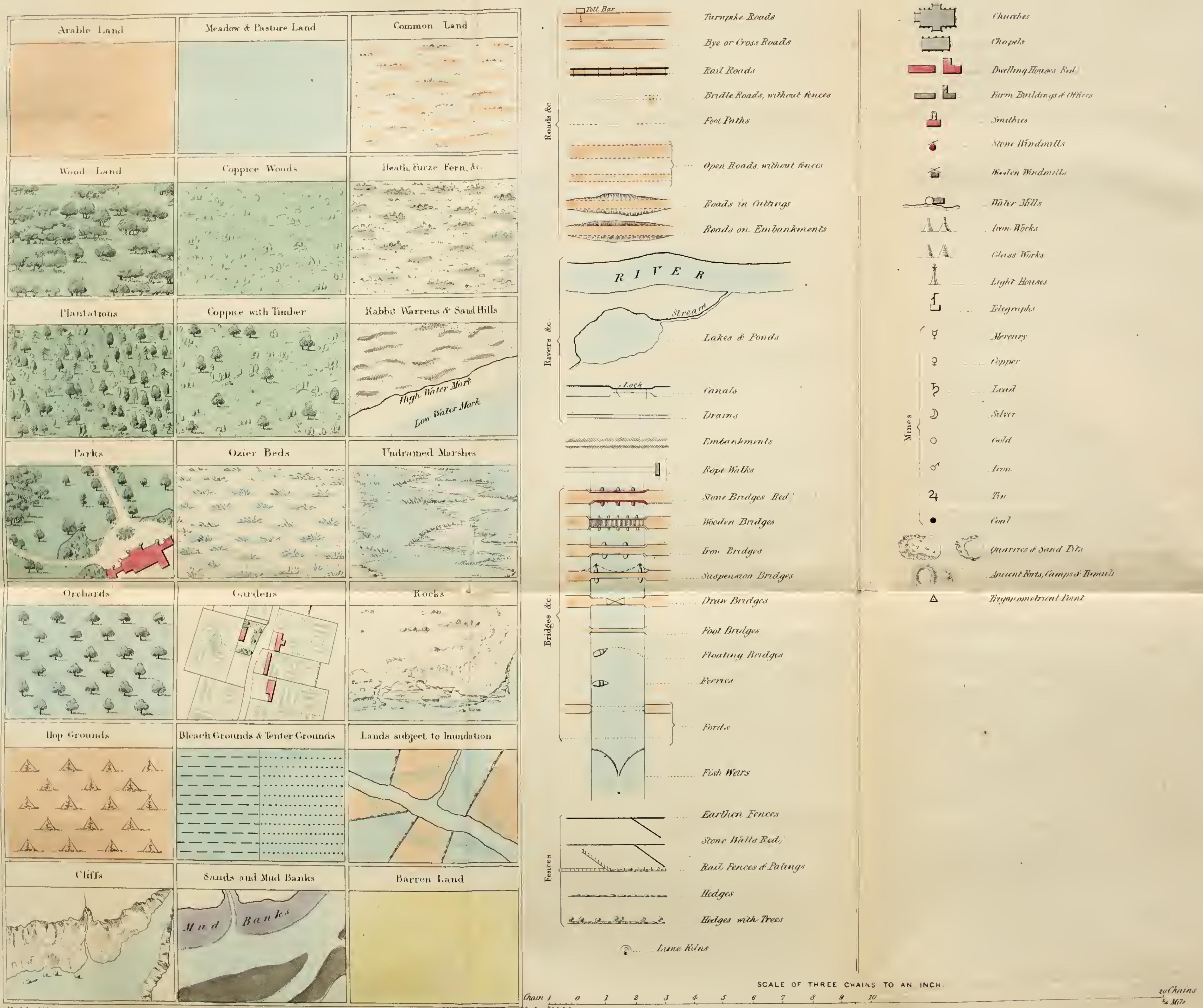

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THE

PRINCIPLES AND PRACTICE

OF

SURVEYING:

LAND AND ENGINEERING,

TRIGONOMETRICAL, SUBTERRANEOUS AND MARINE.

With an Appendix.

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ILLINOIS

BY

CHARLES BOURNS,

SURVEYOR;

MEMBER OF THE INSTITUTION OF CIVIL ENGINEERS.

SECOND EDITION.

LONDON:
JOHN OLLIVIER, 59, PALL MALL.
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a.e.
s.c. Surveying

PREFACE.

For some years past it has appeared strange to me that no sufficient treatise on the subject of Surveying was to be had; and as the necessity for such a thing has of late years much increased, so has my wonder; until at length, during a considerable interval of leisure, I have myself been induced to make an effort at supplying the deficiency.

The extent of my success, or otherwise, remains to be determined by my brethren of the profession. I would however take the liberty of reminding them that this volume is the production of a mere practical man; and that it has been partly written, and wholly printed, during the bustle of active employment; some oversights and many literary inelegances must therefore be looked for, but it is hoped that no vital errors will be found.

The reader is requested to bear in mind the aim with which the volume before him has been written, viz. the formation of a book of reference. To effect which it has obviously been necessary to resort to the labours of others.

Some persons will say there was no occasion to have given the elementary matter; others that it might have been made fuller. My object in this portion has been, to furnish sufficient to refresh the memory. A person taking up the volume to learn these branches would therefore find it deficient in the number of examples; conciseness having been constantly kept in view. It must likewise be borne in mind, that the volume is intended to constitute a consistent whole; so that to understand an advanced part, a person must be conversant with what goes before.

In a treatise like this, an assumption of originality would be absurd, indeed would betray presumptuous ignorance;

but the plan, arrangement and execution, are entirely my own; for to say the truth I have not received even the assistance I expected, and that was not very much, from professional friends. Some, however, have done their best, and to them I return my thanks.

I have found it impossible in most instances to transcribe extracts literally from other works. Some were too verbose, others, in my judgment, required some practical modification; but in all cases the sources of information have been acknowledged; and, to use the words of a distinguished fellow labourer in a neighbouring field, "I hope it will appear that I have duly, yet at the same time, honourably, availed myself of the advantages which they supply." I have also inserted two papers of my own, previously published by the Institution of Civil Engineers, which have been likewise modified. The additional column added to the table of Gradients will be found to be of practical utility.

For reasons already stated, it will be evident that this volume is not adapted for the use of schools; but I would fain flatter myself that it will be found not unworthy the attention of more advanced students; as those in such Universities as have established classes of Civil Engineering, and who will have been previously instructed in the necessary elementary branches.

Considering the number of works on Surveying already in circulation, many persons may deem it hazardous in me to attempt to introduce a new one; if so, I can only state that I was encouraged to undertake the task by the advice of professional friends, and have been confirmed in the resolution by the countenance of the many eminent men whose names appear in the subscription list.

To many readers the Appendix will be not the least interesting portion of this volume.

CHARLES BOURNS.

INTRODUCTION.

An eminent writer has observed that "as a Surveyor has perpetual occasion for calculation, it is necessary that he be familiar with the four first rules of Arithmetic, and the rule of Proportion, both in whole numbers, and in fractions, especially Decimals, with the nature of Logarithms, and the use of Logarithmic Tables; and at least Algebraic notation. As it is his business to investigate and measure lines and angles, and to describe these on paper, he should be well acquainted with the elements of Geometry and Trigonometry, and with the application of these principles to the Measurement of Heights, Distances, and Surfaces. In particular he should be familiar with the best practical methods of solving the ordinary Geometrical problems, and should be expert in drawing lines, and describing figures. He should be acquainted with the principles and practice of Levelling; should know something of the principles of Optics and Magnetism, and should possess at least a smattering of the arts of Drawing and Painting."

Assuming the student to be "familiar with the four first rules of Arithmetic, and the rule of Proportion," it is here proposed to furnish him, chiefly for the purposes of reference, with all the information absolutely necessary for his acquirement in the other elementary branches of knowledge above enumerated. It is hoped, and expected, however, that the student, previous to his taking up this volume, will have become acquainted with some of these subjects; especially Decimal Arithmetic, the Elements of Geometry, and with Drawing. He will, notwithstanding, find it advisable to work his way through the selection of propositions in Geometry here offered; and by no means to neglect carefully to construct the several figures.

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ERRATA.

The Reader is requested to make the following corrections with his pen;—viz.

At the head of page 135, inscribe "PRACTICE OF LAND SURVEYING."—"SECTION IV."

At page 279, number the Section VII, instead of VI.

PRINCIPLES OF SURVEYING.

SECTION I.

Decimal Fractions.

The computations of surveyors are performed to so great an extent by decimal arithmetic, that a knowledge of it is not merely desirable, but absolutely indispensable to them.

Decimal fractions, or substantively, " decimals," are fractions written similarly to whole numbers, but whose values decrease from unity in the same proportion as whole numbers increase from the same point; that is, as whole numbers increase at every figure, from right to left, ten times in value, so decimals decrease at the same rate from the same point. This point is distinguished by a dot (·) or point. It will be evident that too much care cannot be bestowed upon accurately pointing off the decimals, as the whole value depends upon it; placing the dot one figure too far to the right, or too far to the left, may make a difference of millions: thus placing it between each two successively of the following ciphers, will vary the value from mere fifty, to fifty millions.

50.000000000 = 50
500.00000000 = 500
5000.0000000 = 5,000
50000.000000 = 50,000
500000.00000 = 500,000
5000000.0000 = 5,000,000
50000000.000 = 50,000,000

The value of decimals is not altered by the addition of ciphers at the right hand; so that when, as in the above example, they are all ciphers, they may be all expunged; fifty will be as correctly expressed by 50 as by 50.000000.

But as ciphers placed at the right hand of integers increase their value, so ciphers placed to the left hand of decimals decrease theirs:—Thus

.1 denotes one-tenth of unity, say of an acre;

.01 ten times less, or one-hundredth part of unity;

.001 ten times less still, or one-thousandth part;

.0001 ten times less than the last, or one ten thousandth part of unity, and so on; every removal towards the right reducing the value to one-tenth of the preceding value.

ADDITION OF DECIMALS.

The addition of decimals is performed in exactly the same manner as the addition of integers, merely taking care to keep the decimal points under each other, and being careful to retain as many, and no more, decimal places in the sum as the greatest number contained in any one of the items. When the last digit, or that next to the decimal point, amounts to ten, it becomes an integer, and must accordingly be added to the first whole number.

Examples.

.321
.5678
5.3
9.
———
15.1888

456.0000
.2
7.01
.009
.9
2.899
———
467.0180

SUBTRACTION OF DECIMALS.

In subtraction of decimals like care must be taken as in addition, to place the decimal points under each other, and then proceed as in the subtraction of whole numbers.

Examples.

From	59.876	From	532.
Deduct	46.254	Deduct	321.75
	13.622		210.25

In this latter example where there are no decimals in the minuend, it may assist the student to attach as many ciphers to it as there are decimals in the subtrahend.

MULTIPLICATION OF DECIMALS.

In multiplication of decimals, the first operation is similar to that in multiplication of whole numbers; but the product must contain as many decimal places as are contained in both multiplier and multiplicand together; if it does not naturally contain so many, then ciphers must be prefixed to make up the deficiency.

For, the denominator of the decimal part of the product, taken as a vulgar fraction, would be unity, prefixed to as many ciphers as the multiplier and multiplicand contain decimal places; and the decimals in the product must be equal in number to those from which they arise.

Examples.

33.33	4.444	.5555	.3333
2.2	.2	.04	.02
6666	.8888	.022220	.006666
6666			
73.326			

Division of Decimals.

In division of decimals the first process is again similar to that in integers, but the quotient must contain as many decimal places *only* as the number of places in the dividend exceeds the number in the divisor; hence, if the number of places in both be equal, the whole quotient will be a whole number; if the number in the divisor exceeds that in the dividend, then make the number equal, by adding ciphers to the dividend; and when there is a remainder let ciphers be added to it, until it is exhausted, or until as many decimals have been obtained in the quotient as are required; the ciphers so added must, of course, be taken as so many decimal places added to the dividend.

The dividend equals the product of the quotient into the divisor; therefore, if the divisor does not contain as many decimal places as the dividend does, the deficiency must be supplied in the quotient; but if the divisor contains as many decimal places as the dividend, then none are required in the quotient.

Examples.

Divide 5.55 by 1.11	Divide 88.8 by 2.22
1.11)5.55(5.	2.22)88.8(40.
5.55	888
. . .	. . .0

Divide 254796 by 5.44

5.44)254796.000(46837.5
2176
.3719
3264
.4556
4352
.2040
1632
.4080
3808
.2720
2720
....

Reduction of Decimals.

Reduction of decimals is a rule by which the known parts of a given integer are converted into equivalent decimals, and *vice versa.*

To reduce a given vulgar fraction to its equivalent decimal:—

Annex ciphers to the numerator, and divide it by the denominator, the quotient will be the required decimal. For, $\frac{3}{4}$ of one shilling, yard, perch, &c., are equal to one-fourth of three shillings, yards, &c.

Examples.

Reduce $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{11}{64}$ to equivalent decimal fractions.

2)1.0
———
.5

4)3.00
———
.75

64)11.000000(.171875
64
———
.460
448
———
.120
64
———
.560
512
———
.480
448
———
.320
320
———
. . .

A decimal being given, to find its equivalent vulgar fraction, or to express its value in integers of lower denominations.

When the equivalent vulgar fraction is required, place as a denominator, under the given decimal, a unit with as many ciphers as there are decimal places or figures in the given decimal, and reduce the fraction, so expressed, to its lowest terms.

If the value of a given decimal be required in lower denominations, multiply it by the value of its integer in the next lower denomination; then point off in the product, from right to left, as many figures as there were in the given decimal. Multiply the decimal so pointed off by the value of its integer in the next lower denomination; again point off the same number of decimals, and so proceed to the lowest integer, or until all the decimals become ciphers.

Then will the figures to the left of the several decimal points be integers of lower values; and the remainder, if any, a decimal of the lowest value.

Examples.

Required the vulgar fractions equivalent to .5, and to .375 ?

$.5 = \frac{5}{10} = \frac{1}{2}$ $\qquad$ $.375 = \frac{375}{1000} = \frac{3}{8}$

What is the value, in lower denominations, of .528125 of a pound sterling?

.528125
20
10.562500
12
6.7500 Ans. 10s. $6\frac{3}{4}$d.
4
3.00

What is the value, in lower denominations, of .74375 of an acre?

.74375
4 roods
2.97500 Ans. 2r. 39p.
40 perches
39.000

What is the value of .2865 of a shilling.

.2865
12
3.4380 Ans. $3\frac{1}{4}$d. $\frac{752}{1000}$
4
1.7520

ARITHMETICAL DEFINITIONS AND NOTATION.

The term number is applied to an assemblage of units, or to many parts of an assumed unit; unit expresses one whole of a given kind, and may be regarded as the element or base. Thus, speaking of one horse, or one pound, we speak of units; the first of a horse, the second of a pound. But in using the terms six horses, eight pounds, three quarters of an acre, we speak of numbers, in which the first is the unit horse repeated six times; the second is the unit pound repeated eight times; and the third is the fourth part of the unit acre repeated three times.

Numbers formed by the repetition of an undivided unit are called whole numbers, or integers; as ten miles, twenty acres; those which are formed of parts of a unit are called fractions; as two-thirds of a mile, three-eighths of a perch.

The following characters or symbols are used to express results or operations in computation; they are called Algebraic signs.

The sign $+$ (plus or more) denotes addition, and indicates that the numbers between which it is placed are to be added together. Thus $6 + 4$ expresses the sum of 6 and 4, or that 6 and 4 are to be added together.

The sign $-$ (minus or less) denotes that the number placed after it is to be subtracted from the number which precedes it. Thus $8 - 4$ expresses that 4 is to be subtracted from 8.

The sign $\times$ (into) indicates multiplication, and signifies the product of the two numbers between which it is placed. Thus 5×6 denotes 5 times 6, or 30.

The sign $\div$ (by) division, denotes that the number preceding it is to be divided by the number which follows it; the quotient is sometimes expressed in the form of a vulgar fraction. Thus $17 \div 9$ signifies that 17 is to be divided by 9, and the quotient may be represented thus $\frac{17}{9}$.

The sign = two short parallel lines, signifies equality. Thus $8 + 4 = 12$, shows that 8 added to 4 are equal to 12.

Colons and double colons are used between quantities to signify their proportionality. Thus $4 : 8 :: 9 : 18$, denotes that 4 are to 8 as 9 are to 18.

The extraction of roots is indicated by the sign $\sqrt{}$; if any other than the square root is meant, a figure is placed over the sign to express the degree of the root. Thus $\sqrt{8}$ means the square root of 8, $\sqrt[3]{64}$ the cube root of 64, $\sqrt[4]{16}$ the biquadrate or fourth root of 16, and so on.

Involution, or powers of quantities, is denoted by a small figure; thus 6^2 means the square of 6, or 6 multiplied by itself; 4^3 means the cube or third power of 4, that is 64; 6^4 the fourth power of 6 or 1296, and so on.

These characters, although most frequently employed in Algebra, and the higher departments of mathematics, yet may very properly be made use of whenever they secure brevity without sacrificing perspicuity.

MEASURES OF LENGTH.

7.92 inches = 1 statute link.
8.88 ,, = 1 Scotch link.
10.08 ,, = 1 Irish or plantation link.
12 ,, = 1 foot.
3 feet = 1 yard.
3.08 ,, = 1 ell Scotch.
$5\frac{1}{2}$ yards = 1 statute perch.
6 ,, = 1 woodland perch.
$6\frac{1}{6}$ yards, or 6 Scotch ells = 1 fall, or Scotch perch.
$6\frac{1}{4}$,, = 1 Cunningham perch.
7 ,, = 1 plantation, church-land, or Irish perch.
8 ,, = 1 forest perch.

22 yards, or 4 perches, poles, or rods = 1 statute chain.
74 feet, or 4 falls = 1 Scotch chain.
40 perches or poles = 1 furlong.
8 furlongs or 1760 yards = 1 statute mile.
,, or 1972 yards = 1 Scotch mile.
,, or 2240 yards = 1 Irish mile.
3 miles = 1 league.
$2\frac{3}{4}$,, (English) = 1 French league.
$3\frac{2}{3}$,, ,, = 1 Spanish league.
$\frac{3}{4}$,, ,, = 1 Russian verst.
$\frac{11}{12}$,, ,, = 1 Italian mile.
$3\frac{1}{4}$,, ,, = 1 Dutch mile.
4 ,, ,, = 1 German mile.
$69\frac{1}{13}$,, ,, (nearly) = 1 degree of a great circle of the earth.

Measures of Surface.

144 square inches = 1 square foot.
9 square feet = 1 square yard.
$9\frac{1}{2}$ square feet = 1 square ell, Scotch.
$30\frac{1}{4}$ square yards = 1 square statute perch.
36 square ells = 1 square fall, or perch, Scotch.
49 square yards = 1 square plantation perch.
40 square perches = 1 rood.
1210 square yards = 1 statute rood.
1960 square yards = 1 plantation rood.
4840 square yards = 1 statute acre.
7840 square yards = 1 plantation, or Irish acre.
4 roods, or 10 square four-pole chains, or 160 square perches = 1 acre.
640 acres = 1 square mile.

Table of Railway or other Gradients.

						of an inch.		of a foot.	
1 ft.	per mile	=	1 in	5280	=	.15	or	.0125	per chain.
2	,,	=	,,	2640	=	.30	,,	.0250	,,
3	,,	=	,,	1760	=	.45	,,	.0375	,,
4	,,	=	,,	1320	=	.60	,,	.0500	,,
5	,,	=	,,	1056	=	.75	,,	.0625	,,
6	,,	=	,,	880	=	.90	,,	.0750	,,
7	,,	=	,,	754.2	=	1.05	,,	.0875	,,
8	,,	=	,,	660.0	=	1.20	,,	.1000	,,
9	,,	=	,,	586.6	=	1.35	,,	.1125	,,
10	,,	=	,,	528.0	=	1.50	,,	.1250	,,
11	,,	=	,,	480.0	=	1.65	,,	.1375	,,
12	,,	=	,,	440.0	=	1.80	,,	.1500	,,
13	,,	=	,,	406.1	=	1.95	,,	.1625	,,
14	,,	=	,,	377.1	=	2.10	,,	.1750	,,
15	,,	=	,,	352.0	=	3.25	,,	.1875	,,
16	,,	=	,,	330.0	=	2.40	,,	.2000	,,
17	,,	=	,,	310.6	=	2.55	,,	.2125	,,
18	,,	=	,,	293.3	=	2.70	,,	.2250	,,
19	,,	=	,,	277.9	=	2.85	,,	.2375	,,
20	,,	=	,,	264.0	=	3.00	,,	.2500	,,
21	,,	=	,,	251.4	=	3.15	,,	.2625	,,
22	,,	=	,,	240.0	=	3.30	,,	.2750	,,
23	,,	=	,,	229.5	=	3.45	,,	.2875	,,
24	,,	=	,,	220.0	=	3.60	,,	.3000	,,
25	,,	=	,,	211.2	=	3.75	,,	.3125	,,
26	,,	=	,,	203.1	=	3.90	,,	.3250	,,
27	,,	=	,,	195.5	=	4.05	,,	.3375	,,
28	,,	=	,,	188.6	=	4.20	,,	.3500	,,
29	,,	=	,,	182.1	=	4.35	,,	.3625	,,
30	,,	=	,,	176.0	=	4.50	,,	.3750	,,
31	,,	=	,,	170.3	=	4.65	,,	.3875	,,
32	,,	=	,,	165 0	=	4.80	,,	.4000	,,

						of an inch.		of a foot.	
33	ft. per mile	=	1 in	160.0	=	4.95	or	.4125	per chain.
34	,,	=	,,	155.3	=	5.10	,,	.4250	,,
35	,,	=	,,	150.8	=	5.25	,,	.4375	,,
36	,,	=	,,	146.6	=	5.40	,,	.4500	,,
37	,,	=	,,	142.7	=	5.55	,,	.4625	,,
38	,,	=	,,	138.9	=	5.70	,,	.4750	,,
39	,,	=	,,	135.4	=	5.85	,,	.4875	,,
40	,,	=	,,	132.0	=	6.00	,,	.5000	,,
41	,,	=	,,	128.8	=	6.15	,,	.5125	,,
42	,,	=	,,	125.7	=	6.30	,,	.5250	,,
43	,,	=	,,	122.8	=	6.45	,,	.5375	,,
44	,,	=	,,	120.0	=	6.60	,,	.5500	,,
45	,,	=	,,	117.3	=	6.75	,,	.5625	,,
46	,,	=	,,	114.8	=	6.90	,,	.5750	,,
47	,,	=	,,	112.3	=	7.05	,,	.5875	,,
48	,,	=	,,	110.0	=	7.20	,,	.6000	,,
49	,,	=	,,	107.7	=	7.35	,,	.6125	,,
50	,,	=	,,	105.6	=	7.50	,,	.6250	,,
51	,,	=	,,	103.5	=	7.65	,,	.6375	,,
52	,,	=	,,	101.5	=	7.80	,,	.6500	,,
53	,,	=	,,	99.6	=	7.95	,,	.6625	,,
54	,,	=	,,	97.8	=	8.10	,,	.6750	,,
55	,,	=	,,	96.0	=	8 25	,,	.6875	,,
56	,,	=	,,	94.3	=	8.40	,,	.7000	,,
57	,,	=	,,	92.6	=	8.55	,,	.7125	,,
58	,,	=	,,	91.0	=	8.70	,,	.7250	,,
59	,,	=	,,	89.5	=	8.85	,,	.7375	,,
60	,,	=	,,	88.0	=	9.00	,,	.7500	,,

TABLE OF OFFSETS FOR SETTING OUT RAILWAY, OR OTHER CURVES.

Radii in Chains.	Offsets.		Radii in Chains.	Offsets.		Radii in Chains.	Offsets.	
	Ft.	In.		Ft.	In.		Ft.	In.
5	13	2.4	29	2	3.3	70	0	11.3
6	11	0.0	30	2	2.4	75	0	10.6
7	9	5.1	31	2	1.6	80	0	9.9
8	8	3.0	32	2	0.8	85	0	9.3
9	7	4.0	33	2	0.0	90	0	8.8
10	6	7.2	34	1	11.3	95	0	8.3
11	6	0.0	35	1	10.6	100	0	7.9
12	5	6.0	36	1	10.0	110	0	7.2
13	5	0.9	37	1	9.4	120	0	6.6
14	4	8.6	38	1	8.8	130	0	6.1
15	4	4.8	39	1	8.3	140	0	5.6
16	4	1.5	40	1	7.8	150	0	5.3
17	3	10.6	42	1	6.9	160	0	4.9
18	3	8.0	44	1	6.0	170	0	4.6
19	3	5.7	46	1	5.2	180	0	4.4
20	3	3.6	48	1	4.5	190	0	4.2
21	3	1.7	50	1	3.8	200	0	3.9
22	3	0.0	52	1	3.2	220	0	3.6
23	2	10.4	54	1	2.7	240	0	3.3
24	2	9.0	56	1	2.1	260	0	3.0
25	2	7.7	58	1	1.7	280	0	2.8
26	2	6.5	60	1	1.2	300	0	2.6
27	2	5.3	65	1	0.2	320	0	2.5
28	2	4.3						

TABLE TO REDUCE THE HYPOTHENUSAL MEASUREMENT OF AN INCLINED PLANE TO ITS EQUIVALENT HORIZONTAL DISTANCE.

Angle.	Links.	Angle.	Links.	Angle.	Links.
4°.0	0¼	16°.15′	4	25°.50′	10
6 .0	0½	17 .15	4½	26 .30	10½
7 .0	0¾	18 .15	5	27 .10	11
8 .0	1	19 .10	5½	27 .45	11½
9 .0	1¼	20 .0	6	28 .20	12
10 .0	1½	20 45	6½	28 .55	12½
10 .45′	1¾	21 .35	7	29 .30	13
11 .30	2	22 .20	7½	30 .5	13½
12 .45	2½	23 .5	8	30 .40	14
13 .30	2¾	23 .45	8½	31 .15	14½
14 .0	3	24 .30	9	31 .45	15
15 .10	3½	25 .10	9½		

INVOLUTION AND EVOLUTION.

A *power* is a quantity produced by multiplying any given number a certain number of times by itself; this is called involution; but the first power, or the square of the given number, is the only one that concerns the Surveyor, as such. The simple number is called the radix, or root. Thus $3 \times 3 = 9$; here 9 is the square of 3, and 3 is the square root of 9; $9 \times 3 = 27$; 27 is the cube of 3, and 3 is the cube root of 27. $.25 \times .25 = .0625$; $.5 \times .5 = .25$, and $.25 \times .5 = .125$. Hence it is evident that although the powers of whole numbers increase progressively in value, yet the powers of proper fractions decrease progressively.

The extraction of roots, or evolution, is the converse of involution.

Any power of a given number can be found exactly; because we can multiply it by itself; but the root of a given number cannot always be found exactly; thus we cannot exactly assign the square root of 8; but by means of decimals we may approximate to any required degree of accuracy.

SQUARE ROOT.

To extract the square root of any given number, begin at unity, and divide the number into periods of two figures each, both to the right hand and to the left, if there be decimals; if the number of decimal places be odd, make it even by adding a cipher; if the number of figures in the integer be odd, the period last pointed off will consist of only one figure.

Find the greatest square in the first period, that is in the one last pointed off on the left, and place its root as a quotient; having set the square under the first period, proceed as in division; but instead of bringing down one figure to annex to the remainder, bring down one period, that is, two figures; and for a divisor double the quotient, or the root first found, and see how many times it is contained in the new dividend, exclusive of its right hand figure; and set this quotient figure in both the quotient and the divisor. Proceed with the new divisor and the quotient figure as in division again; to the remainder annex the succeeding period for a dividend; double the quotient for a divisor; and so on until all the periods have been brought down. The quotient will consist of as many figures as there are periods in the dividend. If there is a remainder after the given figures have been exhausted, the operation may be continued in decimals, at pleasure, by adding ciphers, in periods of two each.

Example I.

Find the square root of 12345678987654321.

1,23,45,67,89,87,65,43,21(111111111
1

21	.23
1	21
221	.245
1	221
2221	.2467
1	2221
22221	.24689
1	22221
222221	.246887
1	222221
2222221	.2466665
1	2222221
22222221	.24444443
1	22222221
222222221	.222222221
	222222221

. . . .

EXAMPLE II.

Find the square root of 171.0864.

1,71.08,64(13.08 root
1

23 | .71
3 | 69

2608 | . 20864
| 20864

. . . . .

EXAMPLE III.

Find the square root of 2. to seven decimal places.

2.(1.4142135 root
1

24 | 100
4 | 96

281 | . 400
1 | 281

2824 | 11900
4 | 11296

28282 | . . 60400
2 | 56564

282841 | . 383600
1 | 282841

2828423 | 10075900
3 | 8485269

28284265 | 159063100
| 141421325

. 17641775

EXAMPLE IV.

Find the square root of $\frac{5}{12}$.

$\frac{5}{12} = .41666666$

```
                . 41,66,66,66,66 ( . 64549 &c. root
                  36
                  —
        124 )  . 566
          4      496
        ———    ———
       1285 )  . 7066
          5      6425
       ————    ————
      12904 )  . 64166
          4      51616
      —————    —————
     129089 )  1255066
               1161801
               ———————
               . . 93265
```

N.B.—The best mode of doubling the root, for each new divisor, is to add the last quotient figure to the last divisor, as in the foregoing examples.

DRAWING INSTRUMENTS.

It will be necessary for the young surveyor to provide himself with some drawing instruments; a small case will be sufficient to begin with. It is not deemed needful to describe particularly the common instruments, as parallel rulers, compasses, drawing pens, &c., but some general observations will not be out of place.

It will be found to be exceedingly difficult to apply short parallel rulers with any reasonable degree of accuracy; those of less than eight inches in length are, to the surveyor, worse than useless. In laying down a large survey, a ruler of eighteen inches, or even two feet in length, should be used. For common purposes a ruler twelve inches in length, and running on rollers, is very convenient; these rulers are sometimes edged with ivory, which is divided into inches and tenths; but this division is more ornamental than useful.

If the joints of compasses are kept so stiff as to require much force in working, it will be found very difficult to set them, with accuracy; on the other hand, if the joints are too slack, the angle of the legs will be liable to alteration from slight accidental causes; a little attention will soon enable the beginner to keep the joints at a fair working-degree of tightness. Good compasses are generally made with two holes in a small plate, or head of the rivet, and the rivet works in a nut at the other side of the joint, by which, with an appropriate turner, the joints may be adjusted. A surveyor should be provided with compasses of different sizes, so that he may not be obliged, at any time, to stretch the legs too wide apart, lest they slip, or run into the paper, and so the distance be inaccurately laid down, or the paper be disfigured. The points should be kept straight and fine, and the leg should be set in the same place as seldom as possible; for instance, in taking the radius of a circle, the leg should be set in the periphery, and not in the centre. Distances should always be set out by compasses, and in one length, as errors are unavoidably incurred, and multiplied by *stepping* distances. Lengthening bars, or lengthening tubes, are used to extend the application of common compasses; but when an extensive survey is to be laid down, it will be found necessary to have recourse to beam compasses; these and the other instruments are so well described by Mr. F. W. Simms,* and others, and as no description can convey to the mind so correct an understanding of the matter as the instrument itself, the young surveyor is referred to Mr. Simms's work, and merely here recommended to keep his beam compasses carefully in a case when not in use, in a dry and cool situation, and always to prove them previous to using them. To prove them, apply the points to an accurate ivory scale, and if the reading on the scale of the beam does not correspond with that on the applied scale, the instrument requires adjustment.

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* "Treatise on the principal Mathematical Instruments," &c.

Beams are made sometimes of brass, and sometimes of wood, and of various lengths, and the instruments finished with various degrees of nicety; but a beam of well seasoned mahogany, of two feet in length, constructed by a good maker, will be sufficient for ordinary purposes.

Proportional compasses will frequently be found useful in reducing plans.

Straight edges, or rulers for drawing long lines, are generally made of metal, either steel or brass; brass is best, being less liable to deflection; but that metal being so much softer than steel, the edges require to be more carefully kept. These instruments, especially if made of steel or wood, should be frequently proved, by drawing a fine line accurately with the instrument, on a true surface, and then turning the straight-edge end for end, to the other side of the line, if the edge in this second position does not coincide with the line, the instrument has warped, or been injured, and should not be used without rectification, as serious errors may be incurred in pricking off angles from a crooked line.

PROTRACTORS.

Protractors are of different sorts, adapted for attaining different degrees of accuracy. For common purposes, as for surveys of farms, and for laying down short lines, the common brass semicircular protractor, six inches in diameter, will be found sufficiently accurate. The ivory protractor which generally accompanies cases of instruments, is too liable to error to be employed by the practical man, when any other can be had.

Sometimes it is advisable to construct a protractor on a large scale on the paper on which a survey is to be laid down. This is done by first drawing a line to represent the meridian, and intersecting it with another line at right angles ; with the point of intersection as a centre, a circle is described with as great a radius as may be ; then, without disturbing the compasses, set one leg in the point at which the periphery of the circle cuts the side intended to represent the north of the meridian, and step the compasses round, marking with a pencil the slight puncture made at each step ; then set one leg of the compasses in the east point of the periphery, and step round as before, and the circle will be divided to every thirty degrees.* Now if a line be drawn connecting the N and E points, that is, if the chord of 90°. be drawn, and bisected, a line drawn from the centre, through the point of bisection to the periphery, will bisect the quadrant; and thus, having already divided to 30°., by taking in the compasses the distance from 30°. to 45°., the periphery may be divided to every 15°. If the diameter of the circle be drawn from each 45°. division, it will afford a good means of checking the operation. A 30°. division of the periphery may then, by repeated trials, be divided into three equal parts, each of which parts, of course, representing 10°.; by means of this the circle may be divided to every 5°. The remainder must be done by carefully dividing a 5°. division into single degrees. A protractor of this kind may be made as large as the paper will contain; and one having been carefully divided, may be pricked off on to the other sheets, which being drawn in pencil may afterwards be rubbed out. Great care should be taken to make every puncture truly vertical in pricking off. To avoid errors as much as possible, in laying down, very small spaces should not be taken in the compasses; and by continually taking submultiples, or such spaces as will measure some of those already laid down, a constant and satisfactory check on the work is obtained. It will not be necessary to extend the minor divisions beyond the semicircle; if they are required, they may be found by laying a straight-edge across the circle, and by keeping the edge to an already ascertained point, and to the centre, the opposite division may be marked.

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* The circumference of a circle is supposed to be divided into 360 equal parts, called degrees, and marked by a small circle, as 10°, ten degrees ; each degree is divided into 60 parts, called minutes, marked with a dash or accent, as 25′, twenty-five minutes ; and each minute into 60 seconds, distinguished by two accents, thus 30″, thirty seconds ; and the whole would be thus written 10° 25′ 30″ ; reading ten degrees, twenty-five minutes, and 30 seconds. Angles are measured by the number of degrees, &c., cut off on the circumference of the circle, by the lines which form the angle ; the angular point being taken as the centre of the circle.

The radius of a circle from whence the lines on Gunter's scale are formed, is the chord of 60 degrees on the line of the chords.

We have been thus minute in describing a mode of making a protractor, because the young surveyor could not be better employed than in going through the operation. Protractors of this kind may be made with the centre-piece cut away; and the protractor being laid on the paper prepared for a plan to be plotted on, with the meridian of the protractor carefully set to a meridian line drawn on the paper, the protractor then being kept in its place, either with drawing pins, or good weights, the work may be plotted within the circle. When the work extends to the periphery of the circle, the protractor must be moved to another meridian line drawn through the station mark last laid down; and so on, every time the protractor is found to be in the way of the work. Protractors, similar to these, are in general use for plotting road surveys on the ordnance survey of Ireland, but they are taken from an engraved copper-plate.

The most accurate protractor, and that generally used for laying down important surveys, is the circular one made of brass. This instrument is well described by Mr. Simms. Much care should be taken to keep it in adjustment, which should be proved occasionally by setting the centre of the instrument accurately over a fine puncture in a piece of paper, then by pressing down the arms the two points will be given which ought to be in a right line with that at the centre; if the three points are not in one line, the instrument is out of adjustment. Some protractors of this kind are furnished with an adjusting screw; such as are not so constructed must be put into the hands of a competent maker when out of order.

Use of Paper-Protractors.

Lay a parallel ruler upon the centre, and on the angle or bearing at the periphery, which you want to lay down ; then, keeping the rule parallel, and moving it to any station from which it is required to lay off a bearing, draw a line by the ruler from the station, and the distance laid off upon that line, by the plotting scale and a pair of compasses, will be the station line of the required bearing and distance. Then take the next bearing from the field-book, and find it on the protractor, lay the ruler upon the centre, and on the degree and minute, move it parallel, as before, to the new station, draw the line, and prick off the distance in the same manner; and so on for as many as may be. A practical surveyor who will try this method of using a very large protractor, will soon find that it is preferable to almost any other; it may be divided to almost any degree of minuteness.

It may be well to observe, that one line drawn parallel to another has the same bearing as that line itself from the meridian, whether drawn from the centre of the protractor or from any other part of the paper; so that if a parallel ruler be laid upon the centre, and upon the degrees and minutes at the periphery, where a bearing was taken to a mark from the centre, and if the ruler be moved parallel, to any station you have to lay off the bearing from, it is the same bearing as if laid off from that centre. This may be more evident to the student if he will lay his ruler on his meridian, and, moving it parallel, draw a meridian through each station. He should, also, bear in mind that the centre of the protractor represents the centre of the theodolite.

For descriptions of scales, drawing pens, &c., the learner is again referred to Mr. Simms.

The pentagraph also is well described by him. It should be mentioned, however, that maps can never with any degree of accuracy, be enlarged instrumentally; because errors which are inappreciable, and indeed are unavoidable, on a small scale, are inadmissible on a larger one. When a larger plan is required, the only satisfactory way is to plot the work anew.

GEOMETRICAL DEFINITIONS.

1. Geometry is the science of extension, and consists in a knowledge of the construction and properties of the figures or forms of bodies.

2. A point " has a position in space, but no magnitude;" that is, it is indivisible.

3. A line has length, but no breadth; the extremities of lines are points; and the intersection of one line by another is also a point. A right line is the shortest line that can be drawn from any one given point to another. A curve line continually changes its direction. Hence there can be one kind only of straight lines, but an infinite variety of curve lines.

4. A superficies, or surface, has length and breadth, but no thickness; the extremities of a superficies are lines; the intersection of one superficies by another is also a line.

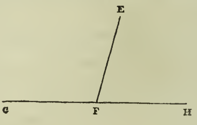
5. A plane is a surface which lies flatly or evenly, between its extremities.

6. An angle is the opening between two lines which meet at a point, called the angular point; that is the greater angle which has the greater opening, or divergence, between the lines or sides by which it is formed, whatever may be the comparative lengths of such lines. A rectilineal, or plane angle, is formed by two right lines; a curvilineal angle by two curve lines. When an angle is expressed by three letters, as A B C, the centre letter is that at the angular point.

7. A right angle is that which is made by one right line falling upon another so that the angles at each side shall be equal. All other plane angles are called oblique angles. When one line falls upon another so that the angles at each side are equal, or right, the lines are said to be perpendicular to each other.

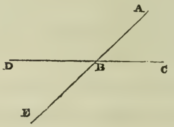
8. Oblique angles are of two kinds, obtuse and acute.

9. An obtuse angle is greater than a right angle, as E F G.



10. An acute angle is less than a right one, as E F H.

11. Two angles, as E F G, and E F H, formed by one line falling upon another, are called contiguous or adjacent angles.



12. Vertical or opposite angles, are those made on contrary sides of two lines intersecting each other, as A B C and D B E.

13. A triangle is a plain figure bounded by three right lines, called the sides of the triangle.

14. An equilateral triangle is one which has three equal sides.

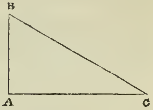
15. An equiangular triangle is one which has three equal angles; and two triangles are said to be similar, or equiangular, when the angles in the one are respectively equal to those in the other.

16. Isosceles, having two sides equal.

17. Or scalene, having three unequal sides.

18. Triangles when described with relation to their angles, are either right angled, obtuse angled, or acute angled triangles.

19. Right angled triangles are those which have one of their angles a right angle, as B A C. The side opposite the right angle is called the hypothenuse, as B C.



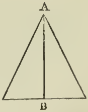
20. Obtuse angled triangles are those which have one of their angles obtuse.

21. Acute angled triangles are those which have all their angles acute.

22. Obtuse and acute angled triangles are more commonly called oblique angled triangles; in all which, any side may be taken as the base, and the other two the legs or sides.

23. Similar triangles are those whose angles are respectively equal, each to each, the sides being proportional each to each, however unequal they may be. Homologous sides are those lying between equal angles in the respective triangles.

24. The height of a triangle is a line, as A B, drawn from any one of its angles, as A, perpendicular to the opposite side, which side then becomes the base; and the angle from whence the perpendicular proceeds is called the vertical angle, or the vertex.



25. Hence all triangles between the same parallels have the same height, since all their perpendiculars will be equal.

26. A quadrilateral or quadrangle is a plane figure bounded by four right lines; it may be either a square, a rectangle, a parallelogram, or a trapezium.

27. A square has its four sides equal, and its four angles right angles.

28. A rectangle, or oblong, has all its angles right angles and its opposite sides equal.

29. Parallel right lines are those which are in the same plane, and so situated that being produced to any extent they will never meet, but continue to be equidistant from each other.

30. A parallelogram is a quadrilateral figure having its opposite sides parallel.

31. A line drawn to the opposite angles of a parallelogram is called a diagonal, or diameter.

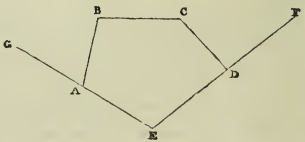
32. A trapezium is a quadrilateral figure which has no two of its sides parallel to each other.

33. A trapezoid is a quadrilateral having only two of its sides parallel.

34. A polygon is a plane figure contained by many right lines. If all the angles and sides are equal, it is called a regular polygon, and denominated according to the number of its sides, as a pentagon, five-sided; a hexagon, six-sided; a heptagon, seven-sided, &c.

35. The internal angles of a figure are those on the inside, made by the lines that bound the figure.

36. An external angle is formed by one side of a figure and the adjoining side drawn out or produced, as it is called; thus C D F and B A G are external angles.



NOTES.

A theorem is a proposition wherein something is proposed to be demonstrated.

A problem is a proposition wherein something is to be done or effected.

A lemma is a demonstration previous and necessary, to render what follows the more evident.

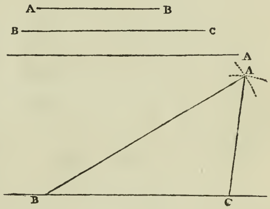
A corollary is a consequent truth, deduced from a foregoing demonstration.

A scholium is a note explanatory of the effect of a foregoing proposition.

PRACTICAL GEOMETRY.

PROBLEM I.

To construct a triangle whose sides shall be respectively equal to three given lines, as A B, B C, C A.

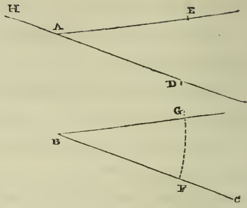


Having drawn a line equal to one of the given ones, say B C, take the distance A B in your compasses, set one leg at one extremity of the line already drawn, and describe an arc; then take the distance C A in the compasses, set one leg at the other extremity of B C, and intersect the arc already described. Now from this point of intersection draw lines to the extremities of B C which will evidently form the triangle required.

It is manifest that any two of the given lines must, together, be greater than the third, as otherwise the arcs described with them would not intersect each other.

Problem II.

At a point B, in a given right line, as B C, to make an angle equal to a given angle, as A.



Take any distance, as B F, but the greater the better within the given line, in the compasses, and with the point B as centre, describe an arc, as G F, intersecting B C; then, with the same distance as radius, and the point A as centre, intersect the sides containing the given angle, as at E and D. Now take in the compasses the distance between the points E and D, found on the sides of angle A, and from the point of intersection F, in the given line, set off on the arc G F, the distance E D; draw a line from B through the point G, which will form the required angle at the given point.

The truth of this is evident, for if the points D and E, and F and G, be connected, we shall have two equal triangles.

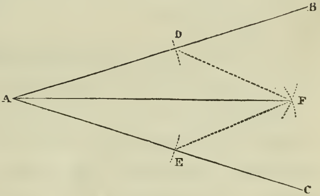
When the given angle is very obtuse, thus if the angle H A E be the required one, it will be advisable to produce one side, as H A, and measure the supplement, D A E, as before; produce the given line, and at the given point B construct an angle equal to this supplement, but in the contrary direction to that for the required angle, which will form the one required, as will be evident from the figure.

Or

With a semicircular protractor; produce the sides of the given angle so that the extremities shall extend beyond the outer edges of the instrument; set the centre of the protractor to the angular point, and the fiducial edge of the diameter to coincide with one of the sides, as A D; note the number of degrees, &c., that is, what portion of the periphery of the protractor is contained within the given angle; then remove the instrument, and set the centre to the point B, making the fiducial edge coincide with the given line, and with a fine needle puncture the paper at the same point of the periphery as was cut by the line A E; remove the protractor, and connect the point B with the point found, and the required angle will be formed.

Problem III.

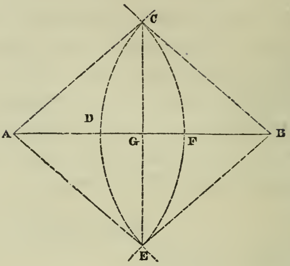
To bisect, or divide into two equal parts, a given right-lined angle, as B A C.



On the lines A B and A C, set off A D = A E, then with any distance more than half of D E, and from those points, D and E, as centres, describe two arcs intersecting each other, as in F; and the right line A F connecting the points A and F will bisect the given angle.

For if D F and E F be drawn, the triangles A D F and A E F will be mutually equilateral, and consequently equiangular; therefore D A F = E A F (Euc. El. prop. viii. B. 1.)

Problem IV.



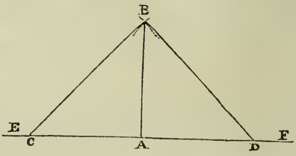
To bisect a given right line, as A B.

With any distance, more than half the given line, as radius, and from A and B as centres, describe two arcs, as C F E, C D E, cutting each other in C and E; connect the points of intersection by drawing the line C E, which will bisect the given line in G.

For let the lines A C, A E, B C, and B E, be drawn, and the two triangles A C E and B C E will be mutually equilateral, consequently the angle A C G = B C G; therefore the triangles A C G and B C G having A C = B C, C G common, and the angle A C G = B C G, the remaining sides A G and G B are equal. (Euc. El. prop. iv. B. 1.)

Cor. Hence it is evident that C E not only bisects A B, but is also perpendicular to it.

Problem V.



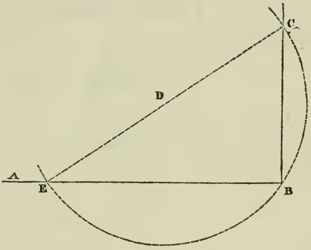
At a given point, as A, in a right line E F, to erect a perpendicular.

From the point A set off at each side the equal distances A C, A D; and from D and C as centres, with any interval greater than A D, describe two arcs intersecting each other, as in B; the line connecting the points A and B will be the perpendicular required.

For let C B and D B be drawn, and the triangles A B C, A B D will be mutually equilateral, and consequently equiangular as in the last; therefore the angles C A B and D A B being equal, are both right angles. (Def. 7.)

For common purposes, as plotting offsets, this is generally done with a plotting scale; thus if a scale be placed on the given line, with one of the divisions on the edge at the given point, and the corresponding division at the other side brought to coincide with the given line, the scale and the line will be at right angles to each other as required.

PROBLEM VI.



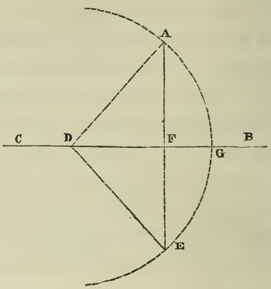
To erect a perpendicular at the extremity, as B, of a given line A B.

From any point, as D, not in the line, with the distance D B, let an arc of a circle be described through B, and cutting A B in any point, as E; draw from E, through D, the right line E D C; and the line connecting the point of intersection C, with the given point B, will be the perpendicular required.

E B C being an angle in a semicircle, is a right angle. (Euc. El. prop. xxxi. B. 3.)

Problem VII.

From a given point, as A, to let fall a perpendicular upon a given right line B C.

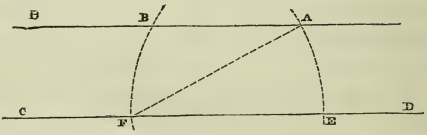


From any point, as D, in the given line, and with D A as radius, describe an arc of a circle, as A G E, take G E = A G, connect the points A and E by the line A F E, and A F will be the perpendicular required.

For D F is a radius, bisecting a chord A E. (Euc. El. prop. iii. B. 3.)

Problem VIII.

Through a given point, as A, to draw a right A B parallel, to a given right line C D.

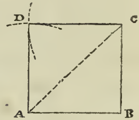


From A to any point, as F, in the given line, draw A F; with the interval F A as radius describe the arc A E, and with the same radius describe B F; take the segment B F = A E, and a line drawn through the points A and B, will be parallel to C D as required.

The angles B A F and E F A being equal (prob. ii.) the lines B A and C D are parallel. (Euc. El. prop. xxix. B. 1.)

This operation is, by the surveyor, generally performed with a parallel ruler; but where great accuracy is requisite, the above mode is preferable.

Problem IX.



On a given line, as A B, to describe a square A B C D.

Make B C perpendicular and equal to A B; and from A and C, with A B as radius, strike two arcs, intersecting each other in D, from whence draw D A and D C, when the required square will be constructed.

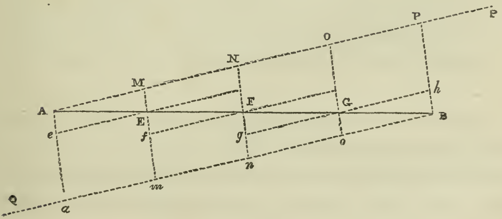
For all the sides are equal by construction; therefore the triangles A B C and A D C are mutually equilateral and equiangular, and, B being a right angle, A B C D is a right angled equilateral parallelogram; for B being right, D is right also; and D A C, D C A, A C B, and B A C are each equal to half a right angle. (Euc. El. prop. xxxii. B. 1.); consequently D A B and D C B will each be a right angle, and A B C D a square. (Def. 27.)

Scholium.

In a similar manner any rectangle, the sides whereof are given, may be constructed.

Problem X.

To divide a given right line, as A B, into any required number of equal parts.

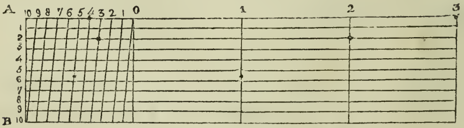


Draw the infinite right line A P, making any angle with A B; and draw B Q parallel to A P; in each of which take as many equal parts A M, M N, &c.; B *o*, &c. as are required; draw A *a*, M *m*, N *n*, &c., and the thing is done.

For let the lines *e* E, *f* F, &c., be drawn parallel to A P, then A M being equal and parallel to *a m*, A *a*, and M *m* are also equal and parallel to each other; and *e* E is equal and parallel to A M; in like manner, M N and *m n*, being equal and parallel, so are M *m* and N *n*: and E *e* and *f* F are equal as well as parallel to both; and so of the other divisions; but A E, E F, &c. are the diagonals of these equal parallelograms, consequently they are equal to each other.

In practice a line may very readily be divided into any number of equal parts by using a scale, finding how many divisions of the scale are contained in the given line, and dividing that number by the number of parts required. For instance, the foregoing figure being measured by a scale of ten; suppose the given line to contain forty divisions, which being divided by four, the required number of parts, gives ten as the length of each part.

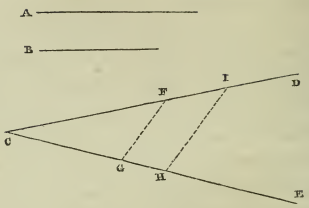
Diagonal scales are divided on the principle of this proposition.



Thus, in the annexed Diagram, let the larger divisions, which are numbered from left to right, represent chains; and let the division on the extreme left be subdivided into ten equal parts; also let the horizontal lines be drawn, dividing the space A B, which is equal to a large division, into ten equal parts. Now if the large divisions, numbered from left to right, each represent one chain, then will the smaller divisions, which are numbered from right to left, each represent ten links, being each equal to one-tenth part of a large division; but, by means of the diagonal lines, each of those smaller divisions is subdivided into ten parts, consequently we are enabled to measure to one link. If each of the large divisions be called ten chains, the smaller ones will be each one chain, and by the diagonal division, we can measure accurately to ten links; any smaller quantity must be computed by the eye. It is to be observed, that in taking off distances the legs of the compasses must be kept parallel to, or at equal distances from, the edge of the scale, otherwise diagonals will be taken; thus suppose the large divisions each to represent one chain, and it is required to take off three chains and forty links; both legs of the compasses must be set in the points dotted in the upper line. If two chains thirty-two links were required, the legs must be set to the points dotted in the second line from the top; when, it will be observed, we have taken the distance required, viz. two chains, reckoned from left to right; thirty links, from right to left, and two links, reckoned vertically, or from the top towards the bottom; all indicated by the respective figures. Let the large divisions each represent ten chains, and the distance required be fifteen chains fifty-five links, that distance will be found to be contained between the two lower dots; that is, we have got ten and five chains, numbered horizontally; and fifty and half of ten vertically.

Problem XI.

To find a third proportional to two given right lines, as A and B.

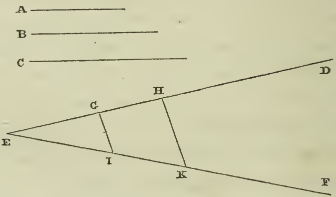


Draw two infinite right lines, as C D and C E, forming any angle. On C D take C F equal to B, and on C E take C G equal to A, and connect the points F and G: again, take C H equal to A, and from H draw H I, parallel to F G; and C I will be the required third proportional.

For C G (= B): C H (= A) : : C F (= A) : C I. (Euc. El. prop. iv. B. 6.)

Problem XII.

Three right lines, as A, B, C, given, to find a fourth proportional.

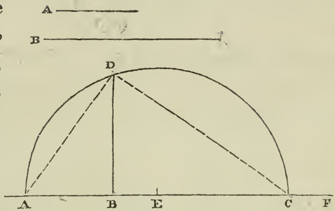


Draw E D and E F, forming any angle; take E G = A, E I = B, E H = C, connect G and I, and draw H K parallel to the connecting line; then E K will be the required fourth proportional.

For E G : E I : : E H : E K; that is A : B : : C : D. (Euc. El. prop. iv. B. 6.)

Problem XIII.

Two right lines, as A and B, given to find a mean proportional.



On an infinite right line, as A F, take A B = A, and B C = B; at B, the junction of the given lines, erect a line perpendicular to A B; bisect A C in E, and with A E as radius, describe the semicircle A D C; then D B will be the mean proportional required.

For let the lines A D, D C be drawn, and the angle A D C will be a right angle, being circumscribed by a semicircle. (Euc. El. prop. xxxi. B. 3.)

The angles A B D, C B D, are right by construction, and the triangles A B D, D B C, are similar; for the angle A B D = D B C, and the angle B D A = B C D; for B A D + B D A equals one right angle (Euc. El. prop. xxxii. B. 1.), and D A C + D C A equal one right angle (same prop.); but B A D and D A C describe the same angle, therefore B D A = B C D; and the remaining angles being equal the triangles are similar; consequently A B : B D : : B D : B C; that is A : B D : : B D : B.*

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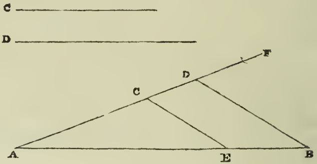
* If the lengths of the lines A and B were given in numbers, the square root of their product would be the mean proportional. For in continued proportion, as it is called, that is when the means, or second and third terms, are equal, their product becomes a square; thus 2 : 4 : : 4 : 8; this is usually written 2 : 4 : 8. Then, as in proportion the product of the extremes, or first and last terms, is equal to the product of the means, it is evident that the square root of the product of the extremes would be their mean proportional; thus 2 × 8 = 16, the square root of which is 4.

Problem XIV.

To divide a right line, as A B, so that one segment thereof, as A E, shall have the same proportion to the whole line A B, as two given lines, C and D, have one to the other.

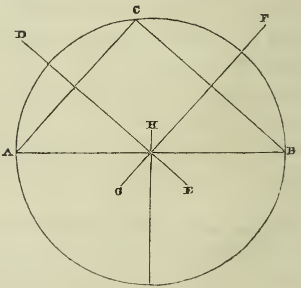


Draw an infinite right line, as A F, to one extremity of the line A B, and making with it any angle; on A F take A C = C, and A D = D; connect the points D and B; and, from C draw C E parallel to D B, then is E the required point of division.

For A C : A D : : A E : A B; that is C : D : : A E : A B. (Euc. El. prop. xv. B. 6.)

Problem XV.

To describe a circle about a given triangle, as A B C.



Bisect the line A C by the perpendicular D E, and C B by the perpendicular F G; then H, the point of intersection of these perpendiculars, will be the centre of the required circle.

By prop. iii. B. 3, the lines D E and F G must each pass through the centre; consequently their point of intersection will be the centre.

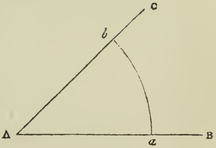
SCHOLIUM.

By means of this problem the centre of a circle may be found, by having a segment only of it given.

PROBLEM XVI.

At a given point, as A, to make an angle of any given number of degrees, say 45°.

From a scale of chords take 60 degrees, and with that interval from A as a centre, describe an arc from the line A B; from the same scale of chords take the required 45°, and set off that distance on the arc, as from *a* to *b*; draw the line A *b* C, and the angle at A will be of 45 degrees, as required.



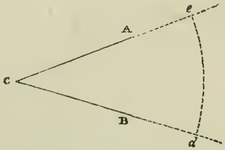
If the required angle were greater than 90° it should be divided into two parts, each less than 90°, and those parts be set off consecutively on the arc.

The required angle may be at once set off with a protractor, vide problem ii.

PROBLEM XVII.

To measure a given angle, as A C B.

If the legs including the angle be not long enough, let them be produced. Thus with a line of chords, if the legs are not so long as the chord of 60°, produce them, and with that chord as radius, and from C as centre, describe the arc, as *e d*; which arc *e d*, measured on the same line of chords, will be the measure of the angle A C B, as required.

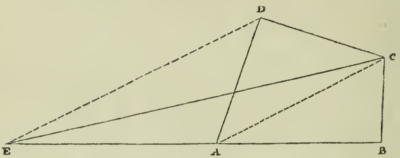


The angle may likewise be measured with a protractor ; vide problem ii.

It may be well to observe, that the centre point of the protractor must always be set to the angular point previous to reading off, or setting off any angle.

PROBLEM XVIII.

To construct a triangle, as E B C, whose area shall be equal to the area of a given quadrilateral figure, as A B C D.

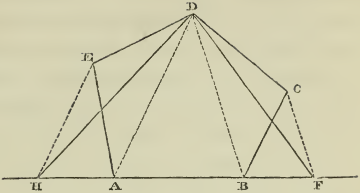


Draw the diagonal A C, and parallel to it a line from D to meet B A produced, as in E; then connect this point of intersection E with the point C, by the right line C E, and E C B will be the triangle required.

For the triangles A D C and A E C, being upon the same base A C, and between the same parallels, (A C and E D), are equal (Euc. El. prop. xxxvii. B. 1.) ; therefore, if the triangle A B C be added to each, then B E C = A B C D.

PROBLEM XIX.

To construct a triangle, as D H F, equal to a given pentagon, or five-sided figure A B C D E.

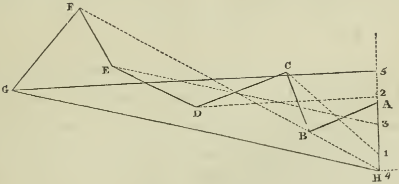


Draw D A and D B, dividing the the given figure into triangles; produce A B to H and F; from E draw E H parallel to A D, and from C draw C F parallel to D B; then connect the points H and F with D, and H D F will be the triangle required.

For the triangle D E A = D H A, and D B C = D F B (Euc. El. prop. xxxvii. B. 1.); therefore, D E A + D B C = D H A + D F B; and if to each side of this equation be added A D B, we shall have D E A + D B C + A D B = A B C D E = D H A + A D B + D F B = D H F.

Equating, or, Give-and-Take Lines.

On the last problem is founded the modes of equating lines and figures, for the purpose of facilitating the calculation of areas.



Let it be required to equate the line A B C D E F G; that is, to draw a straight line which shall represent the crooked one. Produce the line H A; and lay a parallel ruler from A to C, run the ruler down to B, and mark the point 1, where the ruler cuts H A; draw the line C 1; lay the ruler from 1 to D, run it to C, mark on H A the point cut by the ruler, as 2; draw D 2; lay the ruler from 2 to E, run it to D, on H A mark the point 3; lay the ruler from 3 to F, run it to E, mark the point 4; lay the ruler from 4 to G, run it to F, mark the point 5, and draw the line from 5 to G, which will be the one required; so that if the area of the irregular figure, A B C D E F G H were required it might be reduced to the triangle G H 5, the area of which might readily be found from its base and perpendicular. The truth of the construction is evident from the foregoing problem.

In practice it will not be necessary to draw any such lines as C 1, D 2, &c., but it simplifies the operation to the eye of a beginner.

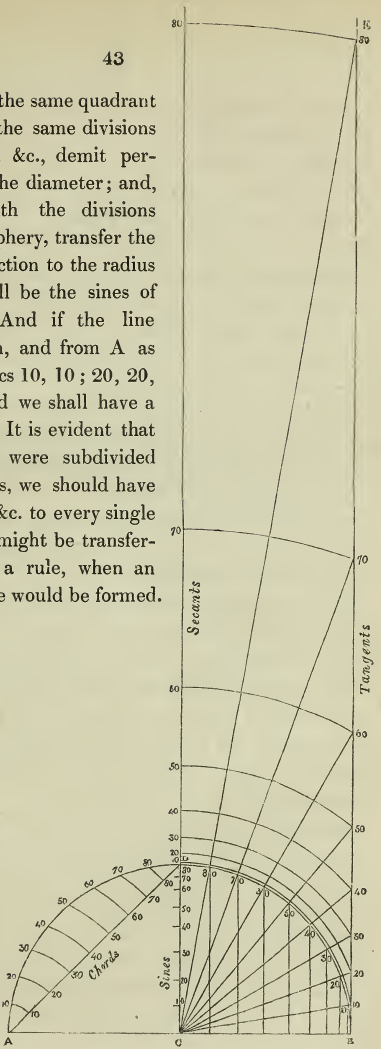
The equating, or balancing of lines, as it is called, is a matter of constant occurrence with the surveyor; the student should therefore make himself thoroughly acquainted with the principle on which it is effected.

Problem XX.

To Project the Lines of Chords, Sines, Tangents, and Secants.

On any line, as A B, describe a semicircle, A D B; from the centre C draw C D, perpendicular to the diameter; and the tangent B E perpendicular to it, at one extremity of the diameter; divide the quadrants, A D, D B, each into nine equal parts, which parts will each represent ten degrees. From the centre C, through every division in B D quadrant, draw lines to intersect B E; we have now obtained the tangents and secants of 10°, 20°, 30°, &c; the secants C 10, C 20, &c. may be transferred to the line C D produced, by describing the arcs 10, 10; 20, 20;

30, 30; &c. In the same quadrant B D, and from the same divisions marked 10, 20, &c., demit perpendiculars on the diameter; and, commencing with the divisions next to the periphery, transfer the points of intersection to the radius C D; these will be the sines of 10°, 20°, &c. And if the line A D be drawn, and from A as a centre, the arcs 10, 10; 20, 20, &c., be described we shall have a line of chords. It is evident that if each division were subdivided to single degrees, we should have sines, tangents, &c. to every single degree; which might be transferred to lines on a rule, when an appropriate scale would be formed.



LOGARITHMS.

When series of terms proceed according to any assignable order, either from greater to less, or from less to greater, by continual equal differences, or by equally increasing or diminishing products or quotients, they are said to be in progression, or to form a progression.

If the terms or quantities proceed by equi-differences, they are said to be in arithmetical progression; but if they proceed by equal multiplications or divisions, they are said to be in geometrical progression.

Thus 0, 1, 2, 3, 4 constitute a series of terms in arithmetical progression; and 1, 10, 100, 1000, 10000 a series in geometrical progression.

Now, logarithms are formed of a series of numbers in arithmetical progression, whose common difference is one; adapted to represent a series in geometrical progression, whose common ratio is ten; hence, the log. of 1 is 0; the log. of 10 is 1; the log. of 100 is 2, &c.; that is, every time a term of the series in geometrical progression is multiplied by ten, the equivalent term of the series in arithmetical progression is increased by one.

Logarithms may therefore be considered the indices of the geometric series. In common logarithms, the geometric ratio being ten, the logarithm of a number is the index of that power of ten which is equal to the said number. Thus, 1000 being the third power of ten, has three for its logarithm. The logarithm of a number comprehended between any two terms of the geometric series, is included between the two corresponding terms of the arithmetic series; that is, the log. of any number between 0 and 10, will be a fraction of unity, &c. When a full term of the geometric series is attained, a full term of the arithmetic series is also arrived at. Thus the logarithm of 10 is 1.00000; of 100 is 2.00000, &c. The integer prefixed to the fractional logarithm is called the Index, or characteristic of the logarithm: and it is evident that the logarithm will have the same index (whether positive or negative) as the less of the two terms of the geometrical series, together with a decimal fraction; the fraction being always positive. The number 50 falling between 10 and 100, its logarithm falls between 1 and 2; it is 1.69897; the index of the less term (10), together with the decimal fraction, which is the log. of 5. The number 500, falling between 100 and 1000, its log. falls between 2 and 3; it is 2.69897; here again we have the index of the less term (100), and the decimal fraction as before.

Practically speaking therefore, the index of the logarithm is always one less in amount, than the number of figures in the natural number. When the natural number is a fraction the index of the logarithm must be negative; thus the fraction .5 which falls between the terms 0. and —10, its logarithm will fall between 0 and — 1; it is indeed —1 + .69897. When the index is negative the sign — is commonly set over it, to distinguish it from the decimal part found in the tables, which, as has been already shown, is always positive: hence the above logarithm — 1 + .69897 is written $\bar{1}$.69897. It is sometimes desirable to reduce the whole expression to a negative form; which is done by making the index less by unity, and taking the arithmetical complement of the decimal; that is, beginning at the left hand, and subtracting each figure from 9, except the last significant one, which is subtracted from 10; the remainder will be the logarithm wholly negative. Thus the logarithm of .5, which is $\overline{1}.69897$, may also be written — .30103, which is entirely negative. Or the logarithm may be made wholly positive by merely prefixing to the tabular decimal the complement of the index to 10; the above logarithm would in that case be expressed by 9.69897; this is increasing the indices in the scale by 10, and is commonly done in the tables of logarithms of sines, tangents, &c. It is convenient in many operations to take the arithmetical complements of common logarithms.

In tables of logarithms of numbers the column of each page signed "N" is that in which a given number is to be sought; and in a line with it, in the adjacent column, is the decimal part of the logarithm of that number. Thus let the logarithm of 365 be required; the adjacent log. in the table is 5622929, to which must be prefixed the index 2, because the number contains three figures, or is between 100 and 1000; the logarithm of 365 is therefore 2.5622929.

Tables of logarithms of numbers generally extend to five figures, sometimes to six, but as specific instructions for using the tables always accompany them, it will not be necessary here to enter into that detail. The logarithms in the tables of numbers are all decimals, and their corresponding natural numbers may be either integers, decimals, or mixt numbers; for the same figures have always the same logarithm, differing only in the index. It is usual, however, to prefix the indices to the logarithms of the numbers from 1 to 100. When the index is negative, in addition or subtraction it will have the contrary effect to the decimal part of the logarithm, viz. in addition the negative index must be subtracted, and *vice versa*. Thus .4234097 is the tabular logarithm of 2651; and, according as the position of the decimal point in the natural

Number.	Logarithm.
2651	— 3.4234097
265.1	— 2.4234097
26.51	— 1.4234097
2.651	— 0.4234097
.2651	— $\bar{1}.4234097$
.02651	— $\bar{2}.4234097$
.002651	— $\bar{3}.4234097$

number is varied, will the index vary. And, as we find the index of a logarithm by the position of the decimal point in the natural number, so, having a logarithm given the position of the decimal point in the natural number will be determined by the index of the logarithm. It must be so placed that the number of integer places may be one more than the index, when positive; or when negative, if the index exceed $\bar{1}$, as many ciphers must be prefixed as will reduce the first significant figure to the decimal place assigned by the index, as shown in the preceding example.

From the construction of Logarithms then, the addition of them answers to the multiplication of their corresponding natural numbers.

And division of natural numbers is performed by subtraction, in logarithms.

To raise the power of a number, multiply the logarithm of the root by the index of the power. And to extract roots, divide the logarithm of the given number by the index of the root required to be extracted.

Any number and its reciprocal have the same logarithm, but with contrary signs; and the sum of the logarithms, of a number and its complement, amounts to 0.

Multiplication by Logarithms.

Add together the logarithms of all the factors, and seek in the tables for the natural number corresponding to the sum; which will be the product required.

Observing, that what is carried from the decimal parts of the logarithms, is so much of an affirmative index. And that the difference between the affirmative and negative indices is to be taken for the index to the logarithm of the product.

Example I.

Multiply 2.76 by 3.29.

The log. of 2.76 is - - -	.4409091
,, 3.29 ,, - - -	.5171959
Product 9.0804 - - -	.9581050

Example II.

Multiply 45.8 by 2.43.

The log. of 45.8 is - -	1.6608655
,, 2.43 ,, - -	.3856063
Product 111.294 - -	2.0464718

Division of Logarithms.

From the logarithm of the dividend subtract the logarithm of the divisor; the remainder will be a logarithm whose corresponding number is the quotient required.

But the index of the logarithm of the divisor must be changed; if affirmative it must be made negative; if negative, affirmative. The sum of the indices, if they be both of the same kind, or their difference if of different kinds, with the sign of the greater, will be the index to the logarithm of the quotient.

When 1 is borrowed in the left hand place of the decimal part of the logarithm, add it to the index of the logarithm of the divisor, when that index is affirmative; or subtract it if negative; then change the index thus found, and work with it as before.

Example I.

Divide 24163 by .4567

Dividend 24163, its log. 4.3831509
Divisor .4567, „ $\bar{3}.6596310$

Quotient 5.290782 . 0.7235199

Example II.

Divide .06314 by .007241

Dividend .06314 . . $\bar{4}.8003046$
Divisor .007241 . . $\bar{5}.8597985$

0.9405061

Here 1, which is positive, carried from the decimals to the $\bar{5}$, which is negative, makes it become $\bar{4}$; and this being taken from the other 4, leaves 0 remaining.

THE RULE OF THREE; OR PROPORTION.

Add the logarithms of the second and third terms together, and from their sum subtract the logarithm of the first, by the foregoing rules; the remainder will be the logarithm of the fourth term required.

INVOLUTION; OR RAISING OF POWERS.

Multiply the logarithm of the given number by the index of the proposed power, and the product will be the logarithm of the power sought.

Example.

Find the 2d. power, or square of 29.

Root 29, its log. is . . 1.4623980
Index 2

Power 841 . . . . . 2.9247960

EVOLUTION; OR EXTRACTION OF ROOTS.

Divide the logarithm of the power, or given number, by its index, and the quotient will be the logarithm of the root required.

Example I.

Find the square root of 39601 ?

Power 39601, its log. is 2)	4.5977062
Root 199	2.2988531

Example II.

Find the cube root of 68921 ?

Power 68921 .. . . 3)	4.8383516
Root 41	1.6127839

LOGARITHMIC SINES, TANGENTS, SECANTS, &c.

By the construction of the figure annexed to problem XX, of Practical Geometry, it is evident that if the radius be supposed any number of equal parts, as 1000 or 10,000, the sine, tangent, &c., of every arc will consist of some number of those parts; some of the tangents and all the secants containing a greater number of such parts than the radius itself; but by computing them in parts of the radius we have tables of sines, tangents, &c., to every arc of the quadrant. These are called natural sines, tangents, &c. And the logarithms of these numbers of parts, which express natural sines, &c. give us tables of logarithmic sines, &c. Thus 9.3836752, the logarithmic sine of 14°, is simply the logarithm of the decimal number .2419219, the natural sine of 14°; radius being 1, say for example

1 mile, 9.3967711 the log. tangent of 14°, is the logarithm of .2493280, the natural tangent of 14°, and so on.

Hutton's tables of logarithms of sines, tangents, &c. from which this description is taken, contain all the sines, tangents, secants, and versed sines, both natural and logarithmic, to every minute of the quadrant; the degrees at the top, and the minutes descending down the left hand side as far as 45°, or the middle of the quadrant; and from thence returning, with the degrees at the bottom, and the minutes ascending by the right hand side, to 90°; thus completing the quadrant. By this arrangement an arc on one side is on the same line with its complement on the other side, and at the same distance from the centre of the page; the respective sines, cosines, tangents, &c. being on the same line with the minutes, and in the columns signed with their several names at top when the degrees are at top, and at bottom, when the degrees are at bottom. The natural sines, tangents, &c. are all placed together in the left hand pages, and the logarithmic ones altogether, facing them, in the right hand pages. In the naturals there are two columns, and in the logarithms three columns, of common differences; each column of differences being placed between the two columns of numbers having the same differences, so that these differences serve for both their right and left hand adjacent columns; and each differential number is set opposite the space between the numbers whose difference it is. The numbers on the same line in those columns which have such common differences, are mutually complements of each other; so that adding the decimals of any two such numbers together, the sum will be 1 integer, leaving no decimal.

It will be observed that in tables of sines, tangents, &c. the indices are expressed, with a separating point between the index and the decimal part; but several of the natural sines, &c. have no index; consequently, when no index is expressed, it is to be understood that all the figures in the column are decimals.

The sine, tangent, or secant of an arc, is expressed by the same number as the sine, tangent, or secant of the supplement of that arc; for this reason the tables are carried only to a quadrant, or 90 degrees. Therefore, when an arc is greater than 90°, subtract it from 180°, and take the sine, tangent, or secant of the remainder, which will be that of the given arc. But this property does not extend to versed sines.

If the young surveyor puts himself fully in possession of what is here written, the use of tables of logarithms will be readily acquired; it is, therefore, not deemed necessary to give examples of taking out logarithms, because they are always given with the tables themselves. We shall, therefore, proceed with trigonometry, in which, only, logarithms are of any real use to the surveyor.

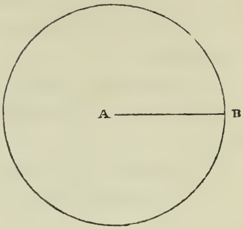
PLANE TRIGONOMETRY.

Plane Trigonometry is that branch of mathematics by which we learn how to compute, or determine, three of the six parts of a rectilinear, or plane triangle, having other three given, when that is possible.

The determination of the mutual relation of the sines, tangents, secants, &c. of the sums, differences, multiples, &c. of arcs or angles; and the investigation of the formulæ connected with them, is also, usually, classed under plane trigonometry.

DEFINITIONS.

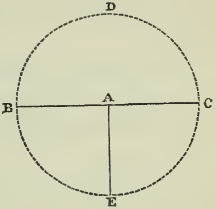
1. A figure is a space enclosed by one or more boundaries. The space contained within the boundary or boundaries is called the area.



2. A circle is a plain figure described by a right line moving about a fixed point; as A B about A; or, it is a figure bounded by one line called the periphery, or circumference, equidistant from a fixed point.

3. The centre of a circle is the fixed point A, about which the line A B moves.

4. A radius is a line A B, that describes the circle.—*Note.* All the radii of the same circle are equal.



5. A diameter is a right line, as B C drawn through the centre from one side to the other; hence the diameter is double the radius.

6. A semicircle is half the circle, as B D C, or B E C, cut off by a diameter.

7. A quadrant, or quarter of a circle, is a part, as B A E, between two radii, perpendicular to each other.

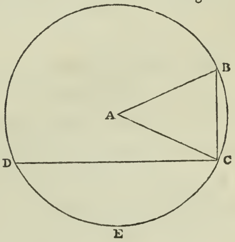


Fig. 25.

8. An arc or arch is any portion of the circumference, as B C (fig. 25.)

2. A chord is any right line, less than a diameter, drawn through a circle from side to side, as D C, and is the measure of the arc. Thus the right line B C is the measure of the arc B C; it is also the measure of the angle B A C.

10. A segment of a circle is a part cut off by a chord, as D B C, or D E C. It may be either greater or less than a semicircle.

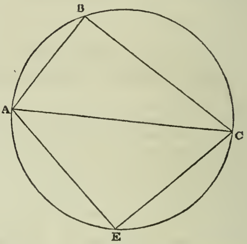
11. A sector is a part of a circle, less than a semicircle, contained between two radii and an arc, as the area B A C, fig. 25.

12. An angle at the centre is one whose angular point is the centre, as A in fig. 25.

13. An angle at the circumference is one whose angular point is in the circumference, as B C E, or D, fig. 26.

14. An angle in a segment is an angle made by two lines drawn from the extremities of the chord of an arc, to any point in the periphery, as B, or E, in fig. 26.

Fig. 26.



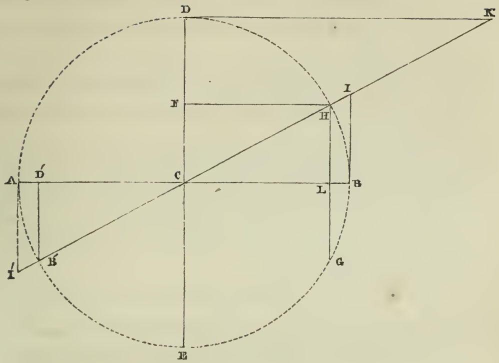
15. The tangent of an arc is a right line which touches the periphery at one extremity of the arc, being perpendicular to the diameter at the same point; and, in Trigonometry, is limited by a right line drawn from the centre of the circle through the other extremity of the arc; thus D K (fig. 27) is the tangent of the arc H D, and A T′ is the tangent of the arc A B′.

Note.—Geometrically, a tangent may be produced at pleasure, to any extent.

16. A secant is the line which limits the tangent; thus, C K is the secant of the arc D H (fig. 27).

17. A sine of an arc is a perpendicular let fall from one extremity of the arc, to a diameter of the circle drawn to the other extremity of the arc; thus H L is the sine of the arc H B.

Fig. 27.



18. The sines on the same diameter increase until they reach the centre, when they become radii. Hence, it is evident, that radius, as C D, is the greatest possible sine, it is thence called the whole sine, or total sine.

19. Since the whole sine, D C, is perpendicular to the diameter, if it be produced to E, the two diameters, A B and D E, bisect each other at right angles; and thus the periphery will be divided into four equal parts, as A D, D B, B E, and E A; and so B D becomes a quadrant or fourth part of the periphery; therefore radius is always the sine of a quadrant.

20. Sines, tangents, &c. are said to be of as many degrees as the arcs of which they are the sines, tangents, &c. contain equal parts of 360; that being the number into which the circumference of the circle is divided; thus radius being the sine of a quadrant, is also the sine of 90 degrees.

21. The versed sine of an arc is that part of the diameter, which lies between the sine and the circumference; thus, L B is the versed sine of the arc G H.

22. The complement of an arc is its difference from a quadrant; and the complement of an angle is its difference from a right angle.

23. The supplement of an arc is its difference from a semicircle; and the supplement of an angle is its difference from two right angles.

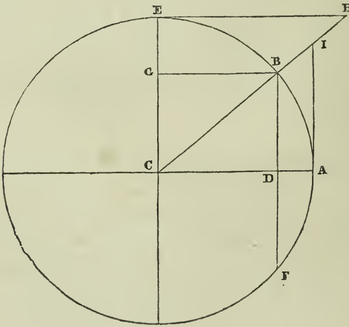
24. The sine, tangent, or secant of the complement of an arc is called the co-sine, co-tangent, or co-secant, of the arc itself; thus, L H (fig. 27) is the sine, B I the tangent, and C I the secant of the arc B H; or the co-sine, co-tangent, or co-secant of the arc H D.

25. The sine of the supplement of an arc, is the same as the sine of the arc itself; for it is the self-same line; thus, H L is the sine either of the arc B H or of the arc H D A.

The following corollaries are deduced from these definitions.—

1. The radius, tangent, and secant, of an arc, as A B, constitute a right angled triangle, as C A I (fig. 28).

Fig. 28.



2. The co-sine, sine, and radius, constitute another right-angled triangle, C D B; similar to the former.

3. And the co-tangent, radius, and co-secant, constitute a third right-angled triangle, H E C, similar to both the preceding.

Hence, when the sine and radius are known, the co-sine is determined by the property of the right-angled triangle.

4. The secant likewise can be determined, having the tangent and radius given, &c. &c.

5. Further, as C D: D B : : C A: A I, it is clear that the tangent is a fourth proportional to the co-sine, sine, and radius.

6. Also C D: C B:: C A: C I; hence the secant is a third proportional to the co-sine and radius.

7. Again C G: G B:: C E: E H; that is, the co-tangent is a fourth proportional to the sine, cosine, and radius.

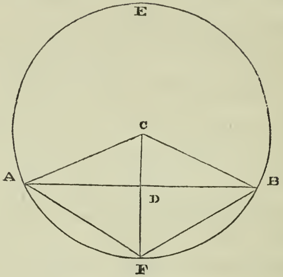
8. And D B: B C:: C E: E H; that is, the co-secant is a third proportional to the sine and radius.

Although the three sides and three angles of a plane triangle, when combined, constitute several varieties, yet they furnish only three distinct cases in Trigonometry, in which separate rules are required.

But before we proceed to the solution of these cases, it is necessary to premise the following Theorems.

THEOREM I.

If from the centre of a circle, A B E, there be let fall a perpendicular, C D, on the chord A B, and produced till it meets the circle in F, the line C F will bisect the arc A B, in the point F.

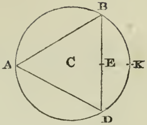


For let the lines A F and B F be drawn; then in the triangles A D F, B D F; A D = B D; D F is common; and the angle A D F = B D F, both being right angles. Therefore A F = F B (Euc. El. B. i. prop. iv.); but in the same circle equal lines are chords of equal arcs, since they measure them (Def. 9); hence the arc A F B is bisected in F by the line C F.

Cor. Hence the sine of an arc is half the chord of twice that arc. For A D is the sine of the arc A F (Def. 17); A F is half the arc, and A D half the chord A B.

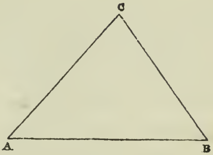
THEOREM II.

In any triangle, as A B D, half of each side is the sine of the opposite triangle.



Let the circle A B D be drawn through the points A, B, D; then the angle D A B is measured by half the arc D K B (Euc. El. B. iii. prop. xx.); that is, the chord B K is the measure of the angle B A D; therefore (by cor. to the last), B E, the half of B D, is the sine of B A D. In the same manner it may be proved that half of each of the other sides is the sine of its opposite angle. Q. E. D.

Theorem III.

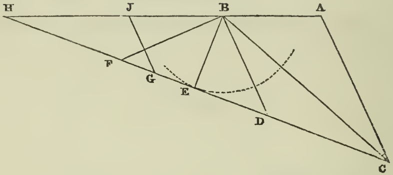


In any plane triangle, as A B C, the sides are proportional to the sines of their opposite angles; that is sin. ang. C : A B : : sin. ang. A : B C; sin. of C : A B : : sin. of B : A C; and sin. of B : A C : : sin. of A : B C.

Half of each side is the sine of its opposite angle, (Theorem ii), but the sines of those angles in tabular parts, are proportional to the same sines in any other measure; therefore the sines of the angles in tabular numbers, will be as the halves of their opposite sides, also in tabular numbers; and as the halves are as the wholes, the sines of angles are to each other, as the sides opposite those angles are to each other; consequently, the sine of any angle is to the side opposite that angle, as the sine of another angle, in the same triangle, is to the side opposite that angle; thus sin. C : A B : : sin. A : B C, &c. Q. E. D.

THEOREM IV.

In any plane triangle, A B C, the sum of two given sides, as A B and B C, including a given angle, is to their difference, as the tangent of half the sum of the two unknown angles, A and C, is to the tangent of half their difference.

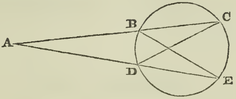


Produce A B, and make B H = B C; from B demit the perpendicular B E, which will bisect the angle H B C (Euclid's El. B. 1. prop. ix); draw B D parallel to A C; make H F = D C, and draw B F; take B J = B A, and J G parallel to B D and A C.

It is evident that A H is equal to the sum, and J H to the difference, of the sides A B, B C; and as B H = B C, and B E is perpendicular to H C, H E = E C, (Euc. El. B. 1. prop. x); and as B A = B J, and J G and B D are parallel to A C, G D = D C = F H; consequently H G = F D, and of course $\frac{1}{2}$ H G = $\frac{1}{2}$ F D, that is E D. Now the angle E B C being half of H B C, is also half the sum of the original angles A and C (Euc. El. B. 1. prop. xxxii.) And as H B, H F, and their contained angle, are respectively equal to B C, C D, and their contained angle; then the angle H B F = D B C = B C A (Euc. El. B. 1. Cor. prop. xxvi); and as angle H B D = A (Euc. El. B. 1. prop. xxix), and angle H B F = B C A, angle F B D is the difference, and E B D half the difference, of the original angles at A and C; then if B E be takenas radius it is evident that E C will be the tangent of half the sum, and E D the tangent of half the difference of the two unknown angles A and C; now J G being parallel to A C; A H : J H : : C H : G H (Euc. El. B. vi. prop. ii), but the halves are as their wholes, therefore A H : J H : : C E : E D; that is, as the sum of the two given sides is to their difference, so is the tangent of half the sum of the two unknown angles, A and C, to the tangent of half their difference. Q. E D.

Theorem V.

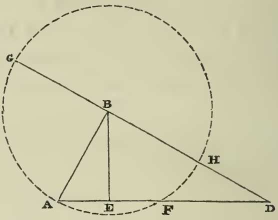
If from a point, A, without a circle, D B C E, there be drawn two lines, A D E, A B C, each of them cutting the circle in two points; the product of one whole line into its external part, viz. A C into A B, will be equal to that of the other line into its external part, viz. A E into A D.



Draw the lines D C, B E; in the two triangles A B E, A D C, the angle A E B = A C D (Euc. El. B. iii. prop. xxi); the angle A is common, and A D C = A B E (Euc. El. B. vi. cor. prop. iv); therefore the triangles A B E, A D C, are mutually equiangular, and consequently A C : A E : : A D : A B (Euc. El. B. vi. prop. iv.) Therefore, A C multiplied by A B, will be equal to A E multiplied by A D. Q. E. D.

Theorem VI.

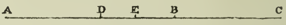
In any right-lined plane triangle, as A B D, the base, as A D, will be to the sum of the other sides, A B, B D, as the difference of those sides is to the difference of the segments of the base, made by the perpendicular B E; that is the difference between A E and E D.



Let D B be produced until B G = B A the less leg; and on B as a centre, with B A as radius, describe a circle, A G H F, cutting B D and A D in H and F. Now it is evident that G D is equal to the sum, and H D to the difference of the sides A B, B D; and as A E = E F (Euc. El. B. iii. prop. xxx), F D is the difference of the segments A E and E D; but A D : G D : : H D : F D (by theorem v.); that is, the base is to the sum of the other sides as the difference of those sides is to the difference of the segments of the base. Q. E. D.

Theorem VII.

If to half the sum of two quantities, be added half their difference, that sum will equal the greater quantity; and if from half the sum be subtracted the half difference, the remainder will equal the less quantity.

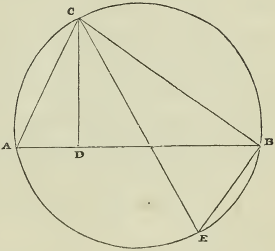


Let the two quantities be represented by A B and B C, forming one right line; A B being the greater, and B C the less; bisect the whole line A C, in E, and make A D = B C; it is evident that A C is the sum, and B D the difference of the two quantities; A E or E C half their sum, and D E or E B half their difference. Hence, if to A E we add E B, we shall have A B, the greater quantity; and if from E C we deduct E B, we shall have B C, the less quantity. Q. E. D.

Cor.—Hence, if from the greater of two quantities, be taken half their difference, the remainder will be half their sum; or if to half their difference be added the less quantity, the sum will be half the sum of the quantities.

Theorem VIII.

If from the point C, of a triangle A B C, inscribed in a circle, a perpendicular be let fall upon the opposite side, A B, that perpendicular will be to one of the sides including the angle C, as the other side including that angle is to the diameter of the circumscribing circle; that is D C : A C : : C B : C E.



Having drawn the diameter C E, connect the points E and B; the angles C E B and C A B are equal (Euc. El. B. 3, prop. xxxi), and C B E is a right angle (B. 3, prop. xxxi), consequently equal to A D C; and the two triangles being thus equiangular the angle E C B = A C D; therefore D C : A C : : B C : E C; or, D C : B C : : A C : E C (B. 6, prop. iv). Q. E. D.

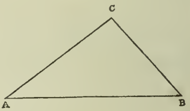
SOLUTION OF THE CASES OF OBLIQUE-ANGLED PLANE TRIANGLES.*

Case I.

When a side and an angle are two of the given parts. As the sides of plane triangles are respectively proportional to the sines of their opposite angles (Theorem iii.), if a side be required, commence the proportion with an angle, or with its sine, which is its representative, and say:

As the sine of the given angle
Is to its opposite side,
So is the sine of one of the other angles,
To its opposite side.

Thus in triangle A B C, let the angle A = 46° 30′, side A B 230, and angle B 37° 30′.



Then as the sum of all the angles = 180° (Cor 1. prop. v.), if the sum of the two given angles, (that is 46° 30′ + 37° 30′ = 84°,) be deducted from 180°, the remainder, viz. 96° equals angle C; but as 96° exceeds a quadrant, we must make use of the supplement to 96°., that is, 84°. (Def. 40), and say (Theorem 1.)

As the sine of C, 96° (or 84°)	9.9976143	
Is to A B . 230		2.3617278
So is the sine of A, 46° 30′		9.8605622
		12.2222900
To B C 167.75		2.2246757

---

* It is generally the best way to work the proportions in trigonometry by means of logarithms, deducting the logarithm of the first term from the sum of the logarithms of the second and third, to obtain the logarithm of the fourth, or required term. Or adding the arithmetical complement of the first term to the logarithms of the other two, we obtain the logarithm of the fourth term.

If an angle be required, commence the proportion, or statement with a side; and say,

As one of the given sides
Is to the sine of its opposite angle,
So is the other given side
To the sine of its opposite angle.

Thus in the above triangle, let the angle C be required, then,

As side B C	167.75	2.2246625
Is to the sine of A, 46° 30′		9.8605622
So is the side A B 230.		2.3617278
		12.2222906
To sine of C, 84° 0′ 1″		9.9976275

Case II.

When two sides and the contained, or included, angle are given.

In the triangle A B C, let A B = 240, A C = 180, and the angle A = 36° 40′; required the remaining side and angles.

Deduct the given angle, 36° 40′, from 180°,* the remainder 143° 20′, is the sum of the sought angles; therefore 71° 40′ is half their sum, and by Theorem iv. the proportion will be,

As the sum of the given sides	240 + 180 = 420.	2.6232493
Is to their difference	60	1.7781513
So is the tangent of half the sum of the unknown angles	71° 40′	10.4796948
		12 2578461
To the tangent of half their difference	23° 20′	9.6345968

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* The three internal angles of a triangle being equal to two right angles, and one right angle containing 90°, the sum of the three angles, of course, equals 180°.

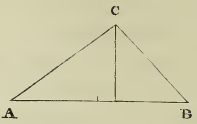


Fig. 34.

To half the sum of the angles .	71° 40′
Add half their difference, now found	23° 20′
The sum is the greater angle .	95 . 00
Subtract half the difference, and	48° 20′ = the less angle.

Then, the angles having been found, the side B C may be had as in the last case, by the following proportion, viz.

Sin. ang. B.		A C		Sin. ang. A.		B C
48° 20′	:	180	: :	36° 40′	:	143.52

Case III.

The three sides given to find the angles.

In the triangle A B C (fig. 34) let A B = 640, A C 470, and B C 340; required the angles A, B, and C.

Assume the longest side, A B, as base, and say (Theorem vi.),

As the base, A B . . .	640 . 2.8061800	
Is to the sum of the other two sides, 470 + 340 . .	810 . . .	2.9084850
So is the difference of those sides	130 . . .	2.1139434
		5.0224284
To the difference of the segments of the base . .	164.53 . . .	2.2162484

Then to half the sum of the segments, that is, to half of the base	320
Add half their difference, now found . .	82.26
Their sum will be the greater segment . .	= 402.26
Subtract half the difference, and we have the less segment	= 237.74

And having let fall a perpendicular from the vertical angle to the point of intersection of the base, the given triangle will be divided into two right-angled triangles, each of which falls under Case I.

RIGHT-ANGLED PLANE TRIANGLES.

Right-angled triangles, as well as oblique-angled, may be solved by means of the rule to the respective case under which any specified example falls; and as a right angle is of course one of the data, it will be found that the rule is generally simplified in its application.

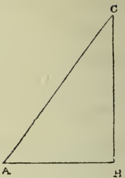
When two sides are given, the third may be found by means of the property in triangles demonstrated in Euclid's Elements, B. 1. Prop. 47.

Thus, hypothenuse^2—the other given side^2 = remaining side^2.

Or, if the hypothenuse be not given, square each of the given sides, and the square root of the sum of those squares will be the hypothenuse.

In the right-angled triangle A B C, let A B = 300, and A C 500.

Hypothenuse, A C 500^2 = 250,000
A B 300^2 = 90,000
4)16,00,00(400
16
·0000

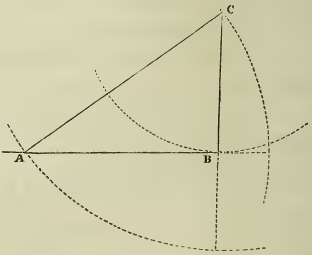


B C = 400, the square root of the difference. Or,

In the same triangle, A B 300^2 = 90,000
B C 400^2 = 160,000
5)25,00,00(500 = A C hypothenuse = the square root of the sum of the squares.
25
··0000

There is another method for right-angled triangles, known by the phrase of "making any side radius." Thus in every right-angled plane triangle, as A B C, if the hypothenuse A C be made radius, and with it as radius an arc of a circle be described from each end of it; A B will evidently be the sine of angle C, and B C the sine of angle A; that is, the legs are the sines of their opposite angles (Defin. 17.)

If B C be made radius, and from C an arc be described with it; then will A B become tangent, and A C secant of the angle C. (Def. 15 and 16.)



If A B be made radius, and, from A as centre, an arc be described, then B C becomes tangent, and A C secant to angle A.

Hence, to find a side. Call any one of the sides radius, and write upon it the word radius; observe whether the other sides become sines, tangents, or secants, and write those words upon them accordingly. Call the word written upon each side the *name* of that side, and say:

As the name on the given side
Is to the given side,
So is the name on the required side
To the required side.

Thus, in the above triangle, if the angles A and C, and the hypothenuse A C were given, to find the legs, the proportion would be

1. Radius : A C : : sine angle A : B C.
2. Radius : A C : : sine angle C : A B.

To find an Angle.

Call any side radius, and proceed as before, but commence the proportion with the given length of the side, thus:

As the given side made radius,
Is to radius,
So is the other given side
To the name on that side,

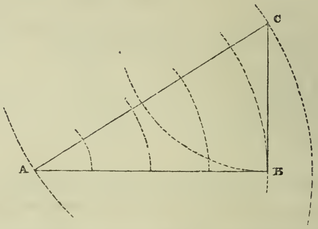
which determines the opposite angle.

Thus, if in the same triangle, the legs were given, to find the angle C; make C B radius, and the proportion will be,

C B : radius : : A B : tangent angle C.

From these proportions it may be observed that, to find a side, any side may be made radius; but to find an angle, one of the given sides must be called radius. And in right-angled, as well as in oblique-angled, trigonometry, to find a side commence the proportion with an angle; and to find an angle commence with a side.

But when the numbers which measure the sides of triangles are under twelve, or are each susceptible of being reduced to factors less than twelve, the solution may be obtained easier without than with logarithms, by the application of the following rule, viz.:



Let A B C be a right-angled triangle; and assuming the base A B, to be radius, B C will be the tangent, and A C the secant of angle A; then if we call radius 1.0000, and divide each of the other sides by it, we shall have the tangent and secant of A in parts of the base taken at 1, that is in parts of 1, to which same radius the table of natural sines, &c. is worked; therefore, making use of that table, to find B C, we have but to multiply the tangent of angle A by the length of A B. Thus let A B = 80 or 8, and angle A = 32° 30.′

Natural tangent angle A 32° 30′ .	.6370703
Multiplied by base A B . . .	80
Length of B C . .	50.9656240

To find A C, multiply the natural secant of angle A, by the length of the base A B. In like manner if B C be made radius, A B becomes the tangent, and A C the secant, of the angle C; therefore the natural tangent and secant of that angle being respectively multiplied by the length of B C, will give the lengths of A B, and A C.

If A C be made radius, then A B will be the sine of angle C, and C B the sine of angle A; therefore, if we multiply the natural sine of angle C, by the length of A C, we shall have the length of A B. And multiplying the natural sine of angle A by A C, will give the length of C B.

The simplicity of these operations arises from the fact of the table being worked to a radius of 1, by which we get rid of one term of the proportion; thus to find C B, making A B radius, we should have, Radius : A B : : tangent angle A : B C; but radius being 1, we have no occasion to divide by it; consequently, multiplying the second and third terms together gives the answer.

This will, perhaps, be more evident to the student if the radius, as A B, be divided into any number of equal parts, say 4, when he will see, manifestly, that the tangent, secant, &c. increase exactly in the same proportion as the radius.

SECTION II.

The Chain.

Gunter's* chain is almost universally made use of by land surveyors in England; it is four perches, making twenty-two yards, in length, divided into 100 links; each link is 7.92 inches long.† At every tenth link a piece of brass is attached, to facilitate the reading of the chain. A surveyor should be particular always to read the chain himself, and never trust it to any other person. The chain should be carefully measured every time it is used; a surveyor will find it convenient to have a chain's length accurately marked on a piece of uniform ground, that so his chain may be proved with little trouble. A chain is necessarily stretched by work, and in consequence is frequently too long when measured; to remedy this, the joints should be examined, and, where open, closed; if this is not sufficient, a ring or two may be taken out; to do this the chain should be doubled, and the two divisions kept as nearly equal as possible. If a chain be found to have shortened, it will have arisen by the bending of some of the links; the remedy is obvious.

A surveyor should always use ten arrows, and see them counted at every change; there is nothing more harassing than to discover the loss of an arrow, without being certain it has occurred within ten chains. He should never go on without finding a missing arrow, or measuring again the whole line on which he is engaged; it being presumed there is no possibility of his having closed the previous line without counting the arrows himself.

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* The Rev. Edmund Gunter, Professor of Geometry, Gresham College.

† To reduce links to feet, deduct one-third of the number, the remainder will be the number of feet very nearly. To reduce feet to links, add one half the number, and the sum will be the number of links, very nearly also Thus, 99—33 links = 66 feet, and vice versa, 66 + 33 feet = 99 links; being within one link of the truth, in the chain.

To make a change, the best way is, when the ten arrows have been used to let the leader measure the eleventh chain; and then hold his handle fast, until the follower advances to him, puts down an arrow, and hands the leader the remaining nine, the surveyor making a cross in his book.

A surveyor should see that the chain-leader draws his chain tight, and puts his arrows down truly, as otherwise three or four inches at every chain's length may very easily be gained or lost. It is considered scarcely necessary to recommend a surveyor to be careful to keep his lines straight.

OFFSET STAFF.

A surveyor should furnish himself with an offset staff. These are commonly ten links in length; but fifteen, or even twenty links, is much more convenient; as such a staff can be more correctly set at right angles to the chain, and an offset be measured in fewer lengths. The staff should be shod with a light iron socket; and be furnished at the other end with a hook (fig. 33), with which bushes may be drawn aside. The links may be marked either by small nails, or by notches. Such a staff, carefully made, furnishes a convenient means of measuring a chain.

SPADE.

A small spade, with one side steeled and sharpened to cut bushes, is very useful. A surveyor should be careful to leave a mark at every station; these should always be, as nearly as possible, of the same shape and size; a distinction being made between principal and secondary ones; thus, suppose at the end of each principal line a cross be cut, at every minor station cut a small triangle.* It is advisable also to drive a good peg at principal stations. In cutting hedges care should be taken to do as little damage as possible; it is unmanly wantonly to injure a man's property in a manner which you dare not do if he was present.

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* The turf cut out should be destroyed or thrown away, otherwise it is very liable to be replaced.

A hook similar to the above affords a good means of pushing the chain through a hedge.

Irish Chain.

In Ireland the chain generally used by country surveyors is only a two-pole chain, divided into fifty links; the old Irish perch being seven yards in length, the chain is fourteen yards, and the link 10.08 inches.

It is evident, that to reduce two-pole to four-pole chains, we should first point off the odd links, then divide the number of chains by two; and if there be one over add fifty to the links. Thus 25.30 two-pole chains equal 12.80 four-pole chains; the four-pole chains, however, being still obviously of the same kind, that is, plantation, Cunningham, &c. as the two-pole ones.*

Scotch Chain.

In Scotland the country chain is seventy-four feet† in length. The measure equivalent to our perch is there called a fall;‡ four of which make a chain.

Some attempts have been made to introduce a chain of greater length into use in England, but they have failed, and are likely to fail again if repeated. Because, in the first place, the present chain is an aliquot part of an acre, and also of a mile; which one of 100 feet, that generally proposed, would not be. The weight of such a chain is objectionable too; because, independently of the fatigue of dragging it, the leading end must be stretched by the great tension.

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* To reduce one measure to another, see Section IV.

† In 1826, when the Scotch standard ell was examined, with a view to establishing the imperial measure, that standard, as well, indeed, as every other, at that time, was found to be inaccurate; which, among other evils, led to discrepancies in the lengths of chains. Some surveyors used chains 74 feet in length, whilst others had them 74.4 feet.

‡ See Table of Measures of Length.

It is to be hoped, however, that the use of the two-pole chain will soon be discontinued; it is theoretically and practically, a bad tool; and as the statute measure is now the legal measure of Ireland as well as of Scotland, there can be no reason for persevering in the use of the short chain, but the prejudice which exists in favour of an old, even though a bad, custom.

ANGULAR INSTRUMENTS.

The Plane Table and Semicircle.

These instruments were formerly much used in England, especially the plane-table, but may now be considered obsolete; descriptions of them are therefore deemed superfluous.*

The Vernier.

The vernier is not an angular instrument; yet, as it constitutes an important part of most of them, must be described in this section. " It is," to use the words of Mr. Simms,† " a contrivance for measuring parts of the space between the equidistant divisions of a graduated scale. It is a scale whose length is equal to a certain number of parts of that to be subdivided, depending on the degree of minuteness to which the subdivision is intended to be carried; but it is divided into parts, which in number are one more or one less than those of the primary scale taken for the length of the vernier; in modern practice, the parts on the vernier are generally one more than are contained in the same space on the primary scale."

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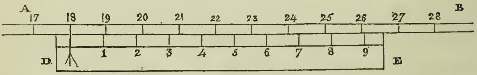
* For descriptions, &c. of them, see Gibson's "Land Surveying;" Wyld's "Practical Surveyor," p. 25, and other old works.

† Treatise on the principal Mathematical Instruments, &c. London, 1836.

The young surveyor will do well to construct a vernier for himself, by means of which he will comprehend its principle much more readily than by any written explanation.

On a piece of fine paper, draw a line, say 10 inches in length, and on it mark the several inches; thus dividing the line into tenths; and on another piece of paper draw a line 9 inches in length, and divide it likewise into tenths of itself; this will readily be done, because each division will be nine-tenths of an inch in length.

Suppose in the accompanying figure, the line A B to represent the limb of an instrument, and D E the vernier.



If the zero, or 0, of the vernier coincide with any division, as 18, on the limb, then is 18 the number of degrees, or whatever they may be, to be set down; but if zero do not coincide with any division, set down the number on the limb next behind zero, as a whole number, and look along the vernier and see which of its divisions does coincide with a division on the limb, and the number of that division on the vernier denotes the number of parts to be set down. Thus in the above diagram, suppose the vernier to be moved back towards the left, until the mark at 5 coincides with that at 22; the measurement would then be 17 and 5-tenths; the whole number being read from the limb, and the parts from the vernier. This will immediately be evident if the limb and the vernier be represented on different strips of paper; it will be found that only one division on the vernier can coincide with a division on the limb, at the same time. It should be borne in mind that the end to be attained is, to ascertain the point on the limb denoted by the zero of the vernier-plate ; thus in measuring an angle, we start from zero on the limb; and the angle is measured by noting what portion of the periphery of the circle is passed over by the zero of the vernier. The means by which this is accomplished, are evident by the foregoing figure; for as each division on the vernier is $\frac{1}{10}$ less than each division on the limb, if the vernier were moved forward one-tenth, the first division of the vernier would coincide with the next succeeding figure, as 19; if moved $\frac{2}{10}$ the second division would coincide with 20, and so on; each division of the vernier coming successively in coincidence with a division on the limb; so that, in this example, when the zero has been moved forward $\frac{2}{10}$, and we consequently require 18.2, the mark at figure 2 will coincide with a mark (20) on the limb. The principle as here xpelained will, it is to be hoped, make its application to practice easily understood.

If it is required to measure to hundredths of an inch the parts of a scale which is graduated to 10ths, it may be done by means of a scale similar to that in the preceding figure. On the limbs of five-inch theodolites, the spaces between the degrees are generally divided into two parts; consequently the short division represents 30′, and the divisions on the vernier are single minutes; because, on the principle already explained, 29 half degrees are divided into 30 equal parts. On seven-inch theodolites the degrees are generally divided to 20′, and the vernier to 20″; the number of subdivisions between the minutes of the vernier being usually the same as between the degrees on the limb; smaller portions must, if required, be estimated by the eye.

The five-inch theodolite is the principal instrument to which this beautiful contrivance is applied in ordinary surveying: a brief description will render its use evident.

It will be observed that some of the divisional lines on the limb are longer than others,* and that those longer ones are numbered from right to left at every tenth; thus 30, 20, 10, &c.; the 360° being the starting point, answering to zero, because the circle being divided all round, the 0 and 360 coincide. The spaces between these lines represent degrees, which are subdivided by shorter lines, each smaller space representing a certain number of minutes; in the case before us 30 minutes. Likewise it will be observed, that some of the divisions on the vernier are longer than others; these indicate single minutes, and are numbered from right to left also; the mark to the extreme right is the zero, or commencement of the index divisions, and is marked Ψ, or Φ. If the zero division of the vernier coincide, or form a straight line, with any line on the limb, that line indicates the required angle, as already explained; if it coincide with one of the shorter lines, as that between 20 and 21, the angle will be 20° 30′. When it happens that the zero division of the vernier does not coincide with any division upon the limb, but stands between two of them, we must observe how many degrees, &c. are denoted by the division it has last passed, and look for a line on the vernier that coincides with one on the limb, and the number of single minutes from that line to the zero of the vernier, added to the number read off upon the limb, gives the required angle. Thus, suppose the zero to stand between 20½° and 21°, and the line on the vernier coinciding with a line on the limb to be numbered 15, then this, added to the former quantity, gives 20° 30′ + 15′; that is 20° 45′, the angle required.

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* The reader is supposed to have an instrument before him.

THE THEODOLITE.

It is necessary for the young surveyor to be acquainted with the adjustments, and the names of the several parts of this important instrument, which is the best used by surveyors for taking either horizontal or vertical angles.

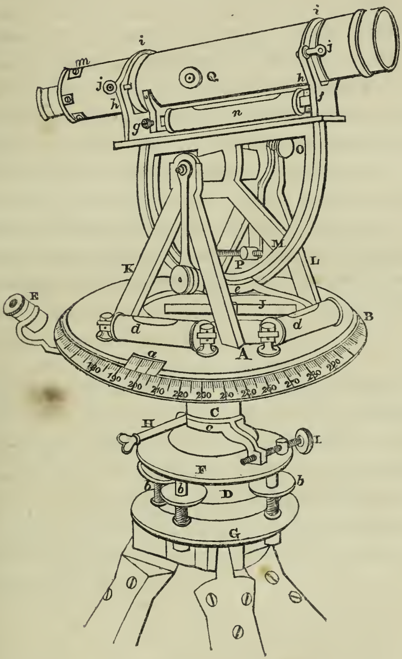
The horizontal limb, A B, see fig. p. 81, consists of two plates, which turn separately on the same double axis; one part of the axis being secured to the upper, or vernier plate, and the other part to the lower, or graduated plate. The axis is conical, and made to work very true. There is a ball-and-socket joint below, at D; and the whole is held together by a strong screw.

The edge of the graduated plate is chamfered off, and generally silvered, to exhibit the divisions. Short portions of the edge of the upper plate are likewise chamfered off to contain the verniers. The instruments in most general use are furnished with two verniers, 180° apart; but those used on the India survey have three verniers; and the larger theodolites are made with four, and sometimes with six. In common instruments the lower limb is generally divided to thirty minutes of a degree, which are subdivided by the vernier into single minutes. When requisite, subdivisions of minutes may be estimated, with the assistance of the microscope, shown at E.

F and G are called "parallel plates," although they never, unless by accident, stand parallel to each other; but by means of the screws, *b b*, connecting them, the instrument is levelled for observation. This is accomplished by turning two opposite screws at a time, one by each hand, in contrary directions; that is, unscrewing one and screwing the other, until the bubble of the small level *d*, which must be brought parallel to the screws in hand, stands in the centre of its run. Then, without disturbing the instrument, the other pair of screws are to be turned in like manner, until their appropriate small level stands in the centre of its run. It is very unwise to use much force with these screws; if one pair work tight, either the other pair should be slacked, or one of the legs of the stand moved, so as to bring the instrument more nearly level, independently of the screws; but, before making any observation, both the small bubbles should stand steadily in the centre of their respective tubes.

The screw H is called a clamp; its use is to retain the head of the instrument steadily in any required position. It acts by tightening the collar, marked C, on the axis. The telescope having been set, by hand, nearly in any required direction, and the clamp H tightened, the cross-hairs may be brought accurately into coincidence with the object by turning the slow-motion screw, I. There is also a clamp to fasten the upper plate to the lower one; there is a screw, called the tangent screw, working with it, horizontally, by which the cross-hairs of the telescope are brought accurately into position when the upper part, only, of the instrument is to be turned.

The surface of one side of the vertical arc, marked M, is divided similarly to the horizontal limb; is read in like manner, and accompanied by a vernier. On the other side a sort of table is sometimes shown, which gives the difference between the hypothenuse and the horizontal side of a right-angled triangle; so that, having set the telescope to the angle of inclination of a hill to be measured, the number of links to be deducted from each chain's length thereof, to reduce it to horizontal measure, may be at once observed on the vertical arc. The level *n*, is adapted for levelling when the instrument is applied to that purpose. The supports of the telescope are called ys; its fastenings, j j, clips.



At the eye-end of the telescope are four screws, adapted for adjusting the diaphragm which carries the cross-hairs composed of spider's thread, and seen in the focus of the eye-glass. These screws are placed, two and two, in lines at right angles to each other; when requisite, they are to be worked by slacking one and tightening its opposite, as much as may be necessary to bring the intersection of the cross-hairs into adjustment.

The focus of the object-glass may be adjusted by means of the screw Q.

A plummet may be suspended from the hook, under the centre of the head of the instrument, for the purpose of placing it accurately over a station mark; but the more usual way with practical surveyers is, having clasped one leg of the instrument, close to the head, with the finger and thumb of one hand, to place a small stone in it with the other, which being let fall from the centre, will denote the point on the ground. If the stone does not fall on the station-mark, the instrument must of course be moved, and the process repeated, until the centre does coincide with the station-mark.

ADJUSTMENTS.

When the theodolite is used for taking angles of elevation and depression, the level marked *n*, under the telescope, must be in adjustment; to ascertain which, let the head of the instrument be levelled by means of the parallel plate-screws, *b b*, then bring the zero of the vertical arc to coincide with the zero of its vernier, when if the bubble of the level *n* stands in the centre of its run, and remains there whilst the upper part of the instrument is turned round on its axis, all is right; but if it does not stand in the centre of its run, open the clips *i i*, and, by means of the tangent-screw P, move the arc until it does; then, taking the telescope carefully out of its Ys, turn it end for end, and replace it, when if the bubble retains its position the level may be considered in adjustment, and the vernier may either be corrected by slacking its screws, moving it until the two zeros coincide, and then tightening the screws again; or, the difference between the zeros when the bubble *n* stands level, may be noticed as an index error; and either added or subtracted, as the case may be, at each observation.

To ascertain the adjustment of the line of collimation, that is, the coincidence of the intersection of the cross hairs with the optical axis of the telescope, set the intersection on some well defined distant point, then turn the telescope one quarter round on its axis, and observe whether or not the intersection of the hairs and the selected point still coincide; if they do, turn the glass round another quarter, and if they still coincide all is right. But if, on turning the telescope, say half-way round, the intersection does not coincide as before, the diaphragm must be moved, by turning two of the opposite screws *m*, slacking one and tightening the other, until, apparently, half the error is corrected; then bringing the points into coincidence again, by the tangent-screw, and turning the glass as before, it will be found that the necessary correction has been made in that direction. The telescope must then be turned one quarter of the way round, and the same point observed; if it coincides with the intersection of the cross-hairs let the glass be turned half round, and the process be repeated, for adjusting the hairs horizontally. If one of these adjustments disturbs the other, the proceeding must be persevered in until the intersection coincides with the same point during an entire revolution of the telescope in its Ys.*

A theodolite having the graduated plate five inches in diameter, will be found to be a very useful instrument for ordinary purposes; but for great accuracy those of larger dimensions must be had recourse to.

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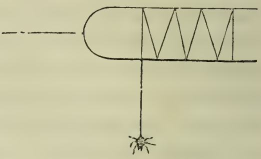
* The adjustments are given here because every surveyor ought to know how to effect them; but the beginner had better not attempt to adjust his instrument himself, lest he do more harm than good.

To replace the cross hairs, if destroyed, is a matter of some nicety. If a spider's thread be used, a piece of wire, in the form of a fork, as recommended by Mr. Wm. Simms, will be found convenient; the prongs being wide enough asunder to admit the diaphragm between them. Having caught a spider, if he can be induced to spin, a thread may be tightly stretched on the fork, and placed on the diaphragm; but spiders are apt to be perverse, or perhaps nervous, and will not always spin when required; in which case a clean web, ready spun, may be selected; those in the open air will be found to be strongest. But a spider's web is so fine and so fragile, that many surveyors will not be able to handle it successfully; it is well, therefore, to be provided with a little floss silk; a thread of which possesses considerable tenacity, and is sufficiently fine for ordinary purposes.



To take out the diaphragm it will be necessary to extract the eight screws which attach it to the telescope tube. Then putting a little fresh varnish, or gum, on the nicks for the hairs, two threads of silk may be readily stretched across. Varnish is preferable to gum, because it is not affected by the humidity of the atmosphere. It must of course be allowed to harden before the diaphragm is replaced.

Having described the theodolite and its adjustments, we may proceed to its application. It may be well to observe, that where much accuracy is desired, the adjustments of the instrument should be examined every time it is taken out; but no practical man will spend his time poking screws about unless they actually require it.

Use.

The theodolite is by very much the best instrument for taking horizontal angles. In commencing a survey, the best system is to set up the instrument, steadily, at the second station, with the vernier at zero, the needle at liberty, and the "N" of the compass-box coinciding with the north end of the needle, both clamps fast, and the instrument truly level. Then slack the upper clamp, and keeping the N side of the compass-box next the eye, direct the telescope to the first or back station; clamp the limbs together, and, if necessary, bring the cross hairs, by the upper tangent screw, accurately to intersect the back station-mark. The reading on the graduated arc will now give the bearing of the first station-line; that is, the angle it makes with the magnetic meridian; (for the telescope coincides with the line, and the needle with the meridian). For although a line connecting the zero points of the two verniers, would cut the line of the telescope at right angles, yet as the telescope and the verniers turn on the same centre, their extremities necessarily pass through equal arcs.

Having noted the reading of the first line, slack the upper clamp, and direct the telescope to the third or forward station; and the verniers will indicate, on the graduated arc, the bearing of the second line.

Read one or both the verniers; but it will be sufficient to set down the degrees, &c. of one, and the minutes, &c. of the other. The mean of such readings, of course, to be plotted.

Now, having ascertained the point under the centre of the instrument, either by a plumb-bob, or by dropping a small stone, carefully, from beneath the centre, let the theodolite be removed to the next station, and there set up, let the lower clamp be loosed, and the telescope brought to bear on the second or back station just left; then re-clamp and level the instrument; see that the cross hairs intersect the pole which has been left in the back station, slack the upper clamp, and turn the telescope on the next forward station; read both verniers as before, which will give the bearing of the third line. Remove the instrument, set a pole in its place, and proceed to the next station, as before; observing always to keep the limbs clamped together, and the vernier at the reading of the last forward station until the instrument has been levelled and the telescope accurately set upon the back station, as that is the bearing of the intermediate station line; the angle merely being read from the opposite sides of the circle.

It is evident that the same diameter of the circle will result if drawn from 180°, as from 360°.

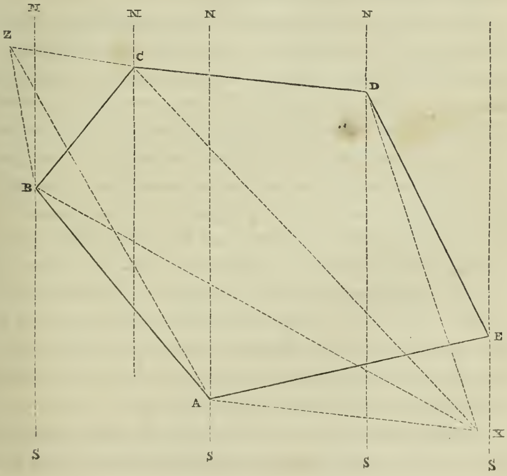
By this method of surveying, generally termed traversing, or surveying by the back angle; the reading on the graduated limb is not the number of degrees contained between the two station lines, commonly called "angles of the field," but the angle that each line respectively forms with the magnetic meridian. Hence, if a paste-board or paper protractor be used, the trouble of shifting it at every angle is avoided, and greater accuracy of plotting is insured.

In measuring an estate or other portion of land, where a circuit is performed, if the work has been done correctly, when the instrument has been set up at every station as above directed, and brought round again to the first, the telescope being directed from thence to the second station, the graduated limb will read the same as it did when first read for that line, for the reason already given. Thus, in the next figure, let A represent the first station, B the second, &c.; then if the instrument be first set up at B, and from thence the bearings of B A and B C be taken, and the bearings of the other lines taken as above directed, to the completion of the traverse round to A, then if the angles have been correctly taken, the bearing of A B taken from

A, will be the same as when read from B. This is called a close; if it does not read the same there is a misclosure.

If the angles are proved to be correct, and yet the work does not close when accurately plotted, it will be evident that the chaining is incorrect.

To avoid complexity, nothing has yet been said about the mode of checking observations, except the closure The most usual way is by intersections; that is, having adjusted the instrument, and read the back angle, or bearing, unclamp the upper limb, direct the telescope to some conspicuous distant object, and set down its bearing, that is, read the graduated limb. Two or three objects may be taken in this way, and if these same objects be taken in the same manner at succeeding stations, lines projected on paper accordingly will intersect each other in the same point. The diagram will render this more intelligible.



Call A the first station; then, the theodolite being adjusted at B, and the reading to A, noted as already specified, direct the telescope to a remarkable object, as Z, which is likely to be visible from some other stations of the proposed survey, note the reading, and lest that object should not be visible at other stations, select a second, as X, and note its reading. Set the vernier to the forward angle and remove the instrument to the next station, where proceed as before, taking as many intersections as are deemed requisite; observing, that at least two bearings are necessary to fix a point; and, until it is fixed, it affords no check. It is evident, for instance, that any two of the above intersecting lines would cut each other somewhere, though the observed angles differed considerably from those projected. Then, as far as the intersections coincide, the work may be presumed to be correct; if they do not coincide, an error has probably been committed at or after the last correct intersection.

The accuracy of the angles being proved, as already directed, and the correctness of the protraction being ascertained, the distance must of course be gone over again on the ground from that point, till the error be discovered. Nothing but practice will enable the student to guess where an error is most likely to have been committed. And here the beginner may be warned to spare no pains in the field; the working of a theodolite appears very complicated to him; nothing is more common than for a learner to forget to clamp the instrument on the forward angle, to turn the wrong tangent screw, or even to neglect to level the instrument. Any such oversight will oblige him to go over all the ground again; he should therefore repeat his angles, that is, read them all over a second time; five minutes thus spent may save him hours afterwards.

Another and a better check is, having started as above directed, the reading of the magnetic meridian line will be the same throughout the survey; that is, having adjusted the instrument at any station, and fixed it on the back station, by clamping the lower limb to the vertical axis, if the work has been so far correctly done, the needle being released, and the graduated limb set at zero, the N and S line of the compass-box will coincide with the needle as at starting. Thus the surveyor has a means of checking his work as he goes on.

ANOTHER METHOD.

By taking angles of the field, that is, the angles contained by each pair of station lines.

This mode, though practised by many surveyors, is certainly not the best, because it affords no check until the work is closed. However, let the instrument be adjusted at the second station, as directed in the preceding article, with the vernier at zero, the lower limb clamped to the vertical axis, the needle at liberty, and the telescope directed to the first station. Read the N end of the needle for the bearing; then let the upper limb be released, and the telescope directed to the third, or forward station; when the reading of the graduated limb is less than 180°, it will evidently be the required angle contained by these station lines, and the reading of the N end of the needle the bearing of the second line. If the reading exceeds 180°, it must be deducted from 360°; thus, if the reading for the back station were 360°, or O, and that for the forward station 300°, it is clear the angle contained by the two lines would be 60°.

The instrument being removed to the next station, the vernier is to be again set at zero, and the telescope directed to the back station, the upper plate then released, the telescope set on the forward station, and the angle read; and so on; thus reading every angle from zero. It is, however, not necessary to read from zero, because the difference of the readings will be the angle required; but, to make the first reading zero is obviously the simplest way, and, therefore, the least liable to error.

TO PROVE THE ANGLES.

By Euc. El. B. 1. cor. prop. xxxii., the sum of the internal angles of a right-lined figure is equal to double as many right angles, minus four, as the figure has sides; therefore, from double the number of station lines deduct four, and the remainder will be the number of right angles equal to the sum of the observed angles, if they have been taken correctly.

In traversing by the first method, it is of no consequence which vernier is read, supposing there are only two; but by the second method the same vernier, that is, the one which has been set at zero, should be read for the degrees.

The magnetic bearing must always be taken from the same end of the needle; that is, having the N side of the box next the eye, take the reading from the north end of the needle.

In measuring a base line for a railway, or other longitudinal survey, the first of the foregoing is manifestly the best method, because it affords a means of checking at any station; whereas the second method does not afford any check unless the work is closed, or the angles are reduced to magnetic bearings.*

If the magnetic bearing be not required when working by the first method, the graduated arc may be set at zero and the telescope brought to coincide with the first station line, which will then become the meridian or datum, as it may be called, from which all the angles are to be plotted. Although convenient, it is of course not necessary, to commence with the limb at zero, but the meridian line on the paper must coincide with that which has been adopted as the meridian on the ground.

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* Vide p. 96.

A theodolite modified by Capt. Everest, has been lately introduced, which, to the civil surveyor has at least novelty to recommend it, as it has three verniers, is generally bronzed, &c. This instrument, its adjustments and uses, are explained and lauded by Mr Simms,* and others; but the degrees must always be read from the same vernier; one pregnant source of error is thus added, which the student will find sufficiently numerous without such addition.

To take Angles of Elevation or Depression.

Having duly levelled the instrument, including, of course, the telescope, observe whether or not the zero of the vertical arc coincides with the zero of its vernier; if it does, it will be necessary merely to bisect an object at the required station by the intersection of the cross-hairs, and then to note the reading indicated on the arc, for the required angle. If the two zeros do not coincide, the quantity of difference must be noted, as an index error, as already mentioned; that is, if when the telescope is level, the zero of the arc stands above the zero of the vernier, the amount of such error of adjustment must be noted, that it may be deducted from the readings of angles of elevation, and added to those of angles of depression. The reason for this will be manifest if the student will bear in mind, that his business is to ascertain the arc through which the axis of the telescope moves. It is scarcely necessary to say, that if the zero of the arc stands below, the error is to be added or subtracted contrariwise.

The Circumferentor.

The circumferentor is an instrument much used in some parts of this kingdom. It is very inferior in accuracy to the theodolite, but being much simpler in construction and in application, much lighter to carry, and much less costly, and less liable to derangement, it will doubtless be continued in use by many for some time to come. It consists commonly of a compass box and magnetic needle, with two brass sights standing perpendicular to the index; in each sight there is a large and a small aperture; in the large aperture a horse-hair, or fine silk thread is fixed; but of late years many circumferentors have been furnished with telescopes. This instrument stands on three legs, similarly to the theodolite. It is connected with its stand by a ball-and-socket joint, furnished with a clamp. The compass-box is generally five or six inches in diameter, and contains two circles of figures; the outer circle is numbered round from right to left in one series of 360 degrees; the inner circle, called the quartered compass, is divided into quadrants of 90° each, reckoned respectively from the N and S points. The instrument is sometimes furnished with small spirit levels; but in general the only means of ascertaining that it is in an horizontal position is, by observing that the ends of the needle are equidistant from the glass lid of the box. As the use of the circumferentor depends entirely on the magnetic needle, it can never be relied on where much accuracy is required; because the needle is subject to a considerable diurnal, as well as annual, change of variation, besides local attraction. The diurnal variation has been observed to vary between 3′ and 20′; being greatest in summer, and least in winter. The maximum on any day is attained to the eastward about 7, A.M., from which time it continues moving westward until between two and three, P.M., when it returns again towards the east.*

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* Principal Mathematical Instruments, &c.

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* Robison's Mechanical Philosophy; Philosophical Transactions, &c.

Local attraction is caused by the proximity of iron to the needle; some ludicrous instances of this have occurred, as, from wire in a wig; spectacles set in steel; a knife in the waistcoat pocket; and even setting up the instrument close to an iron lamp-post; and such inadvertencies are very easily committed. The chain also may be brought too close; and in mines, the tram-plates and tram-waggons, &c. &c.

To Use the Circumferentor.

The quickest mode of using this instrument is to set it up at every alternate station only. Thus, referring to figure 41,* let the circumferentor be set up at station B, carefully levelled, and with the "N" side of the compass box to the eye, direct the sights to station A, and note the readings of the north end of the needle on both series of figures, which will evidently be the magnetic bearing of the first station line; because the needle represents the magnetic meridian, and the line of the sights, which is the zero line of the instrument, represents the station line; and it amounts to the same thing to take the distance of the meridian line from the station line, as that of the station line from the meridian line. The compass-box is marked with the west on the right, and the east on the left-hand side of north, that so the reading of the needle may be the correct reading. Now, without removing the instrument, turn the sights in the direction B C, and note the bearing of that line. The instrument may then be removed to station D, and the bearings of D C, and D E be taken, and so on.

It will be advisable for the surveyor to accustom himself always to read from the same end of the needle, say the north; but in the above case it will be perceived that the bearing of one line is taken from the end, and of the other, from the beginning of the line (so to speak); the line A B, for instance, if taken at B, will bear S E; but if taken at A, would bear N W; this might be remedied by reading from the south end of the needle for the back station, and the north end for the forward station; or by turning the N side of the box to the eye for one station, and the S side for the other; but the safest way is always to read from the same end of the needle, and note accordingly in the field-book,—of which more hereafter.

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* p. 87.

The surest way, however, is, to set up the instrument at every station; and from thence take the bearings of both lines. Thus, if the circumferentor be first set up at B, (see the same figure), the bearings of B A and B C be taken; then removed to C, and the bearings of C B and C D taken; the bearing of C B, from C, will be the converse of the bearing of the same line from B; consequently one reading checks the other. It is evident that the angle contained by any two lines may be found by placing the instrument in the angular point, reading the bearing of each line, and subtracting the less from the greater.

If at any time the needle appears not to play freely on its centre-pin it should be taken out, and the cap and pin carefully cleansed; when the needle has been replaced, direct the sights accurately to a distant object, and note its bearing; then with a knife, key, or other piece of steel or iron, applied to the edge of the box, draw the needle out of its position, and when it again settles, observe whether the object bears exactly as before; if not, either the needle or the pin, or perhaps both, should be replaced.

The Prismatic Compass.

This instrument from its size, is very portable,[*] and being by far the most convenient of angular instruments, may occasionally be used in conjunction with a theodolite or circumferentor, to run secondary lines in a parliamentary survey, but it has not much pretension to accuracy.

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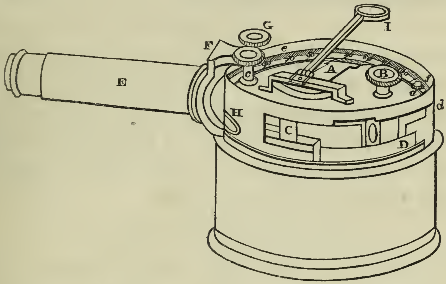
[*] It is described by Mr. Simms.

THE POCKET SEXTANT.

This little instrument may frequently be used with much advantage in the filling up of a parliamentary survey. Some surveyors recommend it for all purposes, except setting out long lines, but the accuracy of such a system is at least questionable. The box-sextant is not, properly speaking, a surveyor's instrument, inasmuch as it does not measure azimuthal angles, unless the objects observed are in the same horizontal plane, which is seldom the case on land: but in surveying coasts and harbours it is an important auxiliary.



The principle of the construction of this instrument is the same as of the sextant, which is well explained by Mr. Simms, to whose work the reader is referred; a description of its uses being deemed sufficient for our purpose. It is represented in fig. 42.

The index is shewn at A; it is moved along the divided limb by a rack and pinion, turned by the milled-head B. At *a* and *d* are supplied means for adjusting the horizon-glass D. A key for making these adjustments generally accompanies the instrument, and fits into a hole made to receive it, as at *c*.

The telescope, E, may, when required, be attached by the screw, G; or the instrument may be used without it, by applying the eye to the orifice H.

The angle is read off in the same manner as on the theodolite.

Having taken the angles of the field, by a theodolite, circumferentor, or other instrument, to reduce them to angles from the meridian, for protraction.

The angles of the field, with the bearing of the first, or any other station line, from the meridian, being given, to find the bearings of the other lines.

Rule I.

If the angle of the field, at any station, be more than 180°., deduct 180° from it, and add the remainder to the bearing of the preceding station-line; the sum, if less than 360°, will be the bearing of the next following line; but if the sum be more than 360°, take 360° from it, and the remainder will be the bearing required.

Rule II.

If the angle of the field be less than 180°, deduct it from 180, and from the bearing of the preceding line deduct the remainder, which will give the bearing of the succeeding or other line; but if the bearing at the preceding station be less than the first remainder, to this foregoing bearing add 360°, and from that sum take the first remainder, and this last remainder will be the required bearing.

Example.

Let the following angles of the field be reduced to bearings of the needle from the north; the bearing of the first station line being 262°.

No. of Station.	Ang. of Field.		
1	159°		
2	200	200—180= 20, 262+20=	282°
3	270	270—180= 90, 282+90=372, 372—360=	12
4	80	180— 80=100, 12+360=372, 372—100=	272
5	98	180— 98= 82, 272—82=	190
6	100	180—100= 80, 190—80=	110
7	230	230—180= 50, 50+110=	160
8	90	180— 90= 90, 160— 90=	70
9	82	180— 82= 98, 70+360=430, 430—98=	332
10	191	191—180= 11, 11+332=	343
11	120	180—120= 60, 343— 60=	283
		180—159 = 21, 283— 21=	262
Sum. =	1620		

The last found bearing being the same as that taken by the needle at the first station, proves that the angles of the field have been truly taken.

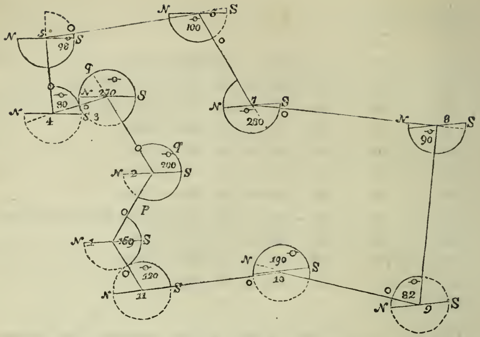
The correctness of the angles of the field may also be proved, as already shewn, by adding them together, and from double the number of station lines deducting 4; the remainder, multiplied by 90°, will be equal to the sum of the angles. Thus,

sides
11 × 2 = 22 — 4 = 18 right angles
90
1620 sum of the angles.

If the angles of the field have been taken without any compass-bearing, one may be assumed for the first station, and the angles reduced for proof, or for protraction.

DEMONSTRATION.

Let the figure 1, 2, 3, 4, &c. represent the plot of the foregoing example; and let the lines N S, passing through every station, represent meridian lines.



The arc NSo, at the first station, will be the bearing of the line 1, 2, from the north, as if taken by the circumferentor, which arc is equal to the arc NrSq at the second station, made by producing the line 1, 2, from the parallel position of the needle. And the arc rSo at the second station—rSq, which is a semicircle, is = oq, but NrSq, which was shown to be equal to the bearing at the first station, + oq = NrSqo, which will be the bearing of the second line.

Thus, if from rSqo = 200°, the angle of the field at the second station, rSq = 180°, be taken, the remainder, oq, will be 20°. But NrSq = 262° the bearing of the first station line + oq, which is 20° = NrSoq = 282°, the bearing of the second station line.

Again, at the third station NrSq = NrSqo, the bearing of the second station line, and rSqNo the angle of the field at the third station; if then, from it the semicircle rSq be taken, the remainder will be qNo, to which, if NrSqo be added, the sum will be the entire circle + No, if therefore the circle, or 360°, be taken therefrom, the remainder No will be the bearing of the third station line, &c.

If the student will take the trouble to protract the figure on a larger scale, and to describe the arcs, the whole will be made manifest.

VARIATION OF THE COMPASS.

The meridian of a station on the earth's surface is a great circle passing through both the poles and through the place; and the intersections of this circle with the sensible horizon of the spectator is called a meridian line, and marks the north and south points of his horizon.

This true meridian line ought to be laid down carefully on every plan intended for a permanent or future reference, as of a county, parish, estate, coal-mine, &c.; because not only does the magnetic meridian vary from the true meridian, but its variation is materially different in different places; and this difference is constantly undergoing changes; nay, almost every magnetic needle has a variation due to itself. This last arises from the difficulty, if not impossibility, of the maker being able at all times so to centre a needle, that the magnetic poles shall coincide exactly with the extremities of that needle. Therefore, where rigid accuracy is required, a true meridian should be marked on the ground, and referred to fixed points on the plan; and not merely to the London magnetic meridian, but to the particular variation due to the needle with which its bearing has been taken. This is a matter too much neglected. Coal and other underground works are, however improperly, almost universally surveyed with the needle; with little, if any, reference to a true meridian. Now, coal-works, particularly, are in general spread over extensive areas, and continue in course of working for many years; during which, not only will different surveyors, and different needles, be employed, but the variation will undergo material changes; thus, in the year, 1576, the variation at London was 11° 15′ E.; in 1622 it was 6° 12′ E.; in 1662 it coincided with the true meridian; in 1700 it was 9° 40′ W.; in 1740 it was 6° 10′; in 1804 it was 24° 8′; in 1815 it was 24° 18′ W.*

Hence it appears that in less than forty years, in one instance, viz. from 1662 to 1700, the variation varied from itself 9° 40′; and this may very reasonably be expected to occur again; for although within the last forty years, little alteration has taken place, yet the laws by which it is governed are as yet unknown to us.

We frequently hear of destructive explosions arising from the workmen "striking into an old working," where there had been an accumulation of gas; and, indeed, this is not so much to be wondered at, as that it does not occur constantly; and that it does not constantly occur can be accounted for only by the surveyors allowing themselves a wasteful space between the workings.

Very vexatious and expensive lawsuits, respecting boundaries, also arise, which may generally be attributed to imperfect surveys made with the needle. Several rules have been given, from which the following are selected, for finding the variation.

1. By equal altitudes.* Having attached a dark-coloured glass to the telescope of a good theodolite, let the horizontal limb be carefully levelled, and observe the sun at some distance to the east of the meridian, that is, say an hour or two before noon, by bringing him on the cross hairs of the telescope. In this position let the horizontal circle be read off, and the vertical arc securely clamped. When the sun has passed the meridian, and is in the descending point of his daily course, let him be followed by moving the upper part of the instrument round to the west, without unclamp-

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* Robison's "Mechanical Philosophy."

† For a mode by Amplitudes, see Lieut. Frome's and other works.

ing the lower limb or the vertical arc, until he comes into the field of view, and until, by continuing the horizontal motion, the sun and the cross hairs once again coincide. In this position, it is evident the sun must have precisely the same altitude above the western horizon, that it had at the moment of its first observation above the eastern. At this point let the motion be arrested, and the horizontal limb be again read off. The difference of the readings will be the azimuthal * or horizontal arc described in the interval. Now it is evident, that when the altitudes of the sun, or of a star, are equal on either side of the meridian, its azimuths or bearings, whether reckoned both from the north, or both the south point of the horizon, must also be equal; consequently, a line through the north and south points of the horizon will bisect the horizontal arc thus determined, and may therefore be marked on the ground. For minute accuracy in observing the sun, either the upper or lower limb should be taken. Also, if much time should elapse between the observations, a correction must be applied for the change in the sun's declination during the interval. From mid-winter to mid-summer the sun gradually approaches the north pole, and, in consequence, a shorter period will intervene between his ascending from a given altitude to the meridian, and his descending from thence to the same altitude on the other side in northern latitudes; and the reverse takes place from mid-summer to mid-winter.

It is advisable to take altitudes at a considerable time from noon, because about noon the sun's motion is nearly parallel to the horizon.

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* Azimuth is the angular distance of a celestial object from the north or south point of the horizon (according as it is the north or south pole of the horizon which is elevated), when the object is referred to the horizon by a vertical circle.—*Herschel.*

It is also advisable to take two or three altitudes, at short intervals of time, as at a quarter of an hour apart, and to note the readings of the vertical arc; lest it should not be possible to take a particular afternoon altitude on account of clouds, &c.

The change of declination is given for every hour, in the first page of each month, in the Nautical Almanac.

It should be borne in mind, that the day is divided into twenty four hours, and the circle of the horizon into 360°. Hence each hour of time measures 15° degrees of space, therefore four minutes of time measure 1° of space; consequently much precision is requisite in astronomical observations.

It is scarcely necessary to say that a true Meridian line being marked on the ground, its bearing by the compass will be the variation, East or West. The variation is said to be East, when the north end of the needle stands at the East side of the true meridian; and West, when it stands at the West side.

2. A meridian line may be marked on the ground without the aid of any instrument, with sufficient accuracy for ordinary purposes, by setting up a pole vertically, on a perfectly horizontal plane or platform. At, say, two hours before noon, measure the length of the pole's shadow, and from the bottom of the pole, as a centre, describe an arc with this distance as radius. Observe when the end of the shadow coincides with the periphery of this arc, about the same time after noon; connect these two points of coincidence of the arc and the shadow, and bisect this chord. If similar observations be made intermediately, as at every half hour, the points of bisection of the chords will all be very nearly in a right line with the pole, and will thus give a true meridian, if the work be carefully done.

The imperfection of this method of finding the meridian arises from the indistinctness of the termination of the shadow of the upright pole or wire, and from the change of the sun's declination between the observations.

4. There is however another as simple and exact as can be expected without the assistance of a Telescope. It is known that the pole star, and the star Alioth,* or ε of the Great Bear, pass the meridian within about nine minutes of each other. If therefore we suspend two plummets by threads, to the ends of a rod, which turns horizontally on a pivot, and by moving the rod keep the two plumb-lines exactly between the eye and Alioth, when near the meridian, then, at the moment when the pole star is also seen along the two plumb-lines, the two stars and the plumb-lines will be all nearly in the plane of the meridian. This will be readily understood, by referring to a celestial globe, and bringing these stars under its meridian. To keep the plumb-lines steady in the vertical plane, the plummets may hang in vessels filled with water. The pole star's right ascension, or time of culminating, is given for every day in the year, in the Nautical Almanac.

If we could determine the position of any star, when its altitude is greatest, or least, we should then have a point of the Heavens in the plane of the meridian, but that position cannot be found with certainty, because the change of altitude is imperceptible to a sensible distance on each side of the meridian.

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* Alioth is, popularly speaking, the first star in the tail of the Bear.

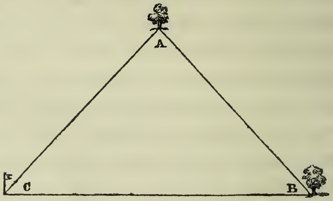
SECTION III.

HEIGHTS AND DISTANCES.

Example I.

Required the distance between two trees A and B, which could not be directly measured, by reason of a large pool of water occupying a considerable portion of the intermediate space.

Measure the distance of each tree from a third object, as a flag-staff, C; let A C = 620, B C = 840 feet; at C take the angle, A C B, between the two trees, = 47° 30′.



Here we have evidently two sides and the included angle given; to find the third side; we must therefore proceed according to Case II of oblique angled-triangles.

$180^\circ - 47^\circ\ 30' = \frac{132^\circ\ 30'}{2} = 66\ 15'$; therefore

As the sum of the given sides	620 + 840 = 1460	3.1643529	
Is to their difference	220	. . .	2.3424227
So is the tangent of half the sum of the unknown angles	66° 15′		10.3565369
			12.6989596
To the tangent of half the difference -	- 18° 54′		9.5346067
Add half the sum of the angles -	- 66 15		
The greater angle, A =	85 .09		

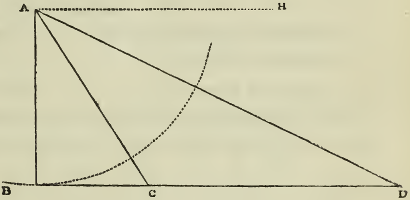
Then

As sine angle A	85° 09′	9.9984422	
Is to B C	840		2.9242793
So is sine angle C	47.30		9.8676309
			12.7919102
To the required distance, A B, 621.54 feet			2.7934680

Example II.

Wanting to know the distance between two inaccessible objects, which lay in a direct line from the bottom of a tower, on whose summit I stood, I took the angles of depression of the two objects; that of the nearest measured 57°, that of the most remote 25° 30′, the height of the tower was 120 feet, what is the distance between the objects.

The figure being constructed as in the margin, the altitude of the tower, A B = 120; and A H being the horizontal line from which the angles have been measured, we have



H A D =25° 30′, hence B A D = B A H—H A D = 64° 30′.

H A C = 57° 0′, hence B A C = B A H—H A C = 33° 0′.

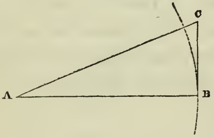
Now if A B be made radius, B D and B C will become the tangents of the respective angles B A C and B A D; and C D the difference of those tangents; therefore multiplying this difference of the natural tangents by the height of the tower we shall have the required distance.*

* Vide Page 70.

Natural tangent - - 64° 30′ 2.0965436
Natural tangent - - 33° 0′ . 6494076
Difference - 1.4471360
Multiplied by height 120
Distance C D = 173.6563200

Example III.

Standing on a horizontal plane, at a distance of 500 feet from the bottom of a tower, the angle of elevation to the top was 21° 48′; required the height.



In this case there is given the distance A B, and the angle A to find B C; then if A B be taken as radius, B C is the tangent of angle A; therefore BC = A B × Natural tangent angle A.

Natural tangent A, 21° 48′ - - .3999715
Multiplied by AB 500
Height of tower BC = 199.9857500 Feet.

When the numbers are large, if logarithms be made use of, it will be, log. BC = log. AB + log. tangent A.

When a number of heights are to be ascertained the following tables (extracted from "Mathematics for practical men," by Olinthus Gregory, LL.D.) will be found useful; observing always to add the height of the instrument to the tabular height.

If angle A = 11° 19′ then BC = $\frac{1}{5}$ AB very nearly
A = 16 42 „ BC = $\frac{3}{10}$ AB „
A = 21 48 „ BC = $\frac{2}{5}$ AB „
A = 26 34 „ BC = $\frac{1}{2}$ AB „
A = 30 58 „ BC = $\frac{3}{5}$ AB „
A = 35 00 „ BC = $\frac{7}{10}$ AB „
A = 38 40 „ BC = $\frac{4}{5}$ AB „
A = 45 00 „ BC = AB „ exactly

Therefore if the instrument be set to one of these angles, and the observer advance or recede until it accord with the elevation of the object, he will have nothing to do but measure the distance between him and the bottom of the object, and the fractional part of such distance, appropriate to the angle in the above table, will give the height.

Thus if the angle of elevation taken at A, were 26° 34′, the distance from A to B 400 feet, and the height of instrument 5 feet, we should have $\frac{400}{2} + 5 = 205$ the height required.

When it is not convenient to measure a station line to the foot of the object;

Let the instrument be set to two of the following corresponding angles successively; that is to say, let it be set to the angle 45°, and find the point C (fig. p. 105), on the horizontal plane, at which the object AB subtends that angle; then set the instrument at 26° 34′, and recede from C, in the direction BCD, till the object has that elevation. The distance CD, between the two stations, will be equal to AB, the height of the object.

Also if the angle at C = 40°, and D = 24° 31½′, CD will = AB
,, if C = 35°, D = 22° 23′, CD = AB
,, if C = 30°, D = 20° 6′, CD = AB
,, if C = 20°, D = 14° 56′, CD = AB

Because in all these cases, the natural cotangent of D — cotangent C = Radius; therefore if AB be made radius, it will be equal to CD, which is the difference of the natural cotangents.

It must be borne in mind, that the heights in all these cases, are those above the point of observation; that is, above the eye of the observer, or more accurately, above the centre on which the telescope turns, that being the angular point.

Example IV.

Wishing to know the height of a spire, to the bottom of which I could not measure, on account of a high wall standing between the church and me, I fixed upon stations which were 93 feet apart, on a horizontal line from the bottom of the spire; and at each station took the angle of elevation to the top; the angle at the nearest station was 55° 54′, at the other 33° 20′. Required the height.

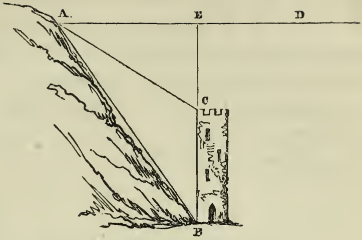
Referring to figure in p. 105, we have, given, the distance CD, and the angles of elevation at C and D. The angle B A D is the complement of angle D, and B A C the complement of C, (Defin. 22); therefore calling A B radius, and A the centre as before; B D and B C are the tangents of those complemental angles, consequently the cotangents of the angles themselves, D and C; and making use of the table of natural sines, &c. and this theorem, viz. $A\,B = \dfrac{CD}{\text{Cot. } D - \text{Cot. } C}$

$$\begin{array}{rlr} \text{We have Cotangt. } D = \text{Cot. } 33^\circ\ 20' = & & 1.5204261 \\ \text{Cotangt. } C = \text{Cot. } 55\ \ 54\ = & & .6770509 \\ \hline & \text{Their difference} = & .8433752 \end{array}$$

$$\text{Hence } A\,B = \frac{93}{.8433752.} = 110.27 \text{ feet, = height required.}$$

Example V.

Desiring to know the height of a tower which stood at the foot of a hill, I measured from the bottom of the tower, up the hill a distance of 250 feet, and at that distance took the angles of depression to the bottom and top of the tower, that to the bottom, viz. D A B = 55° 15′, and to the top, D A C, 31° 15′. What was the height?—

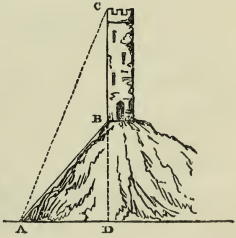


In the triangle A C B we have angle CAB = DAB 55° 15′ — D A C 31° 15′ = 24° 0′, ABE the complement of E A B = 34° 45′, and the side A B = 250; then deducting the sum of the angles C A B and C B A, from 180°, the remainder 121° 15′ = A C B, therefore

Sine A C B	121° 15′		
or its supplement	58° 45′	9.9319213	
Is to A B	250		2.3979400
As sine C A B	24°		9.6095969
			12.0075369
Is to required height in feet		119.02	2.0756156

Example VI.

The height is required of a tower, standing at the summit of a regularly sloping hill, the top and bottom of the object being visible from one station.



The angle to the bottom measures 48° 30′; that to the top 67°; and the distance measured up the hill, to the bottom of the tower, 136 feet. Let the figure be constructed as in the margin, drawing the perpendicular, D C, through the point B; and in the triangle, A B C, there is given, the angle A C D, which being the complement of D A C, = 23° 0′; C A B, being the difference between the two observed angles of elevation, viz., 67°—48° 30′ = 18° 30′; and the side A B = 136. Therefore

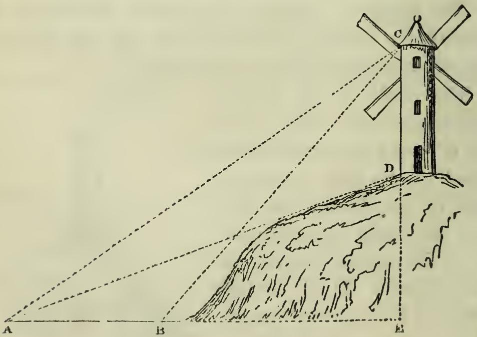
Sine angle C 23° 0′ -	9.5918780	
Is to A B 136 - -		2.1335389
As sine C A B 18° 30′ - -		9.5014764
		11.6350153
Is to required height 110.49		2.0431373

If D B were required, it could readily be obtained by means of the right angle at D.

Example VII.

Requiring the altitude of a windmill, to the bottom of which I was refused access, I measured a distance of 104 feet on the horizontal plane below, and from the extremity A took the angles of elevation to the top and bottom of the mill, viz., to the foot 21° 0′, to the top 35° 0′, and from B the angle to the top 48° 30′.

Construct the figure as annexed, by drawing the infinite right line A E, setting off the distance A B; from A and



B the observed angles; and from C the point of intersection letting fall the perpendicular C E.

The angle C B E—C A B = A C B = 13° 30′, therefore,

Sine A C B 13° 30′ - 9.3681853
Is to A B 104 - - 2.0170333
As sine C A B 35° - - 9.7587717
11.7758050
Is to B C 255.63 - - 2.4076197

Then in the triangle C B E, to find C E.

Radius - - 10.0000000
Is to B C, 255.63, as already found, 2.4076197
As sine of observed angle CBE 48° 30′ 9.8744561
12.2820758
Is to C E - 191.08 - 2.2820758

And in the same triangle, to find B E.

Radius - 10.0000000
Is to B C 255.63 - 2.4076197
As B C E (complement of C B E) 41.30 9.8212646
12.2288843
Is to B E 169.39 - 2.2288843

But A B + B E = A E, that is 104 + 169.39 = 273.39 = A E.

And in the triangle A E D, to find D E, say,

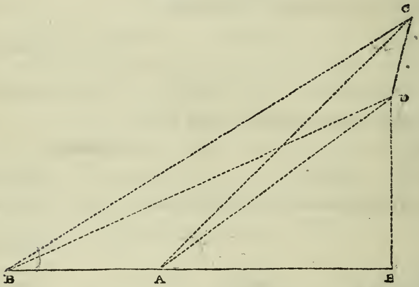
Sine of A D E (complement D A B)
69° 0 - - 9.9701517
Is to A E 273.39 - 2.4367826
As sine of D A E 21° - 9.5543292
11.9911118
Is to D E 104.94 - 2.0209601

And C E—D E = D C, that is 191.08—104.94 = 86.18 = height of mill as required.

Of course there are various other statements by means of which the desired height might be ascertained.

Example VIII.

To find the height of an inaccessible object, as C D, which stood obliquely, on a hill. Having measured the station line A B, of 104 feet in length, from A and B, I took angles of elevation to both the bottom and the top of the object, viz. at A the angle to the bottom measured 36° 30′; to the top 44° 30′; and at B the angle to the bottom = 24° 30′; and to the top 32° 0′.



The figure having been constructed; angle E A C—A B C = A C B = 12° 30′.

In triangle A B C, to find A C, say

Sine A C B : A B :: Sine A B C : A C
12 30° : 104 :: 32° 0 : 254.63

And E A D—E B D = A D B; i. e. 36° 30′—24° 30′ = 12° 0′; therefore in the triangle A D B, to find A D, say,

Sine A D B : A B :: Sine D B A : D A
12° 0′ : 104 :: 24° 30′ : 207.43

C A E—D A E = C A D, *i. e.* 44° 30′—36° 30′ = 8° 0′; therefore in the triangle C A D we have C A = 254.63, D A = 207.43, and angle C A D = 8° 0′, to find D C, (case II, oblique-angled plane triangles). Now deducting the known angle viz. 8°, from 180° *i. e.* the sum of the three angles, the remainder 172° = the sum of the unknown angles.

A C + A D : A C—D A : : Tangent $\frac{1}{2}$ ADC + ACD : Tangent $\frac{1}{2}$ A D C—A C D ; that is

462.06 : 47.20 : : 86° 0′ : 55° 36′.

86° 0 + 55° 36′ = 141.36 = A D C; and 86° 0′—55° 36′ = 30° 24′ = A C D.

Sine A C D : A D : : Sine C A D : D C

30° 24′ : 207.43 : : 8° 0′ : 57.05 = Required height.

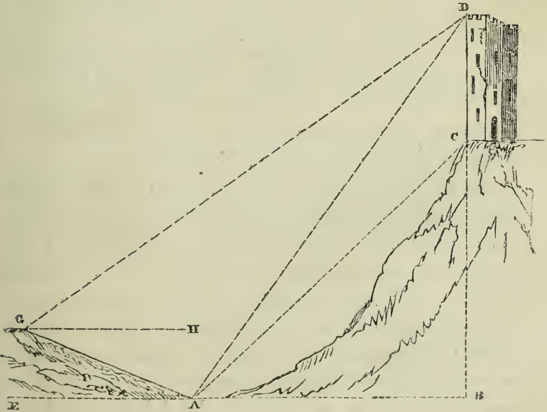
To find D E; in triangle A D E, say

Radius : A D : : Sine D A E : D E

10.000 : 207.43 : : 36° 30′ : 123.38 = Height of hill.

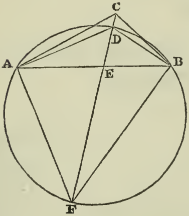
Example IX.

To find the height, C D, of an inaccessible object, on a hill C B, from ground which is not horizontal.



Having measured the station line G A, let the angles H G D, B A D, B A C, and E A G, be taken. G H and E A being parallel, $HGA = EAG$, therefore $EAG + HGD = AGD$; and $180^\circ -$ the sum of E A G and $BAD = GAD$; and $180^\circ - AGD + GAD = GDA$; thus we have found all the angles of the triangle A G D, together with the side A G; from which A D may readily be found, (Case I, oblique-angled plane triangles). The angle $DAB - CAB = DAC$, and $90^\circ - BAD = ADC$; and $180^\circ -$ the sum of D A C and $ADC = ACD$; so we have obtained the several angles, and the side A D, of the triangle A C D; hence C D may be found (Case I, as before). A C having also been found, B C will very readily be obtained.

Example IX.



Standing at a point, F, from whence could be seen three objects, A, B, C, whose distances from each other were known, viz. $AB = 800$, A C 600, and B C 400 yards; I took the horizontal angles $AFC = 33^\circ 45'$, and $BFC = 22^\circ 30'$. It is hence desired to determine the distance of F from each of the stations.

As the distance of F, from any one of the stations, is not given, the figure cannot be constructed in the common direct manner, but having laid down the triangle A B C to a given scale; from A draw A D, making with the line A B an angle of $22^\circ 30'$; and from B, draw B D, making the angle $DBA = 33^\circ 45'$. Describe a circle to pass through the point of intersection D, and through the points A and B. From C, through D, draw a right line to cut the circle again, as in F, which will be the required point. For let F A, and F B, be drawn, and as the angles A F D and A B D stand on the same arc A D, they are equal (Euc. El. prop. xxi, B. iii); and for the same reason $B F D = B A D$. So that at F the angles are of the observed value.

A figure constructed carefully in this manner, will give a good approximation to the desired distances.

To compute the lengths of the sides. In the triangle A B C, the sides being all known, let the angles be found. In the triangle A B D where all the angles are known, and the side A B; find one of the other sides, A D. Deduct B A D from B A C, the remainder, D A C, is the angle included by two known sides, A D and A C; from which the angles A D C and A C D may be found, by theorem IV. Section II. The angle $C A F = 180° — (A F C + A C D)$. Also $B C F = B C A — A C D$; and $F B C = A B C + F B A = A B C +$ supplement of A D C; for in the triangles A E D and F E B, the angle $A E D = F E B$; $D A E = B F E$; consequently the remaining angles, A D E and F B E are equal.

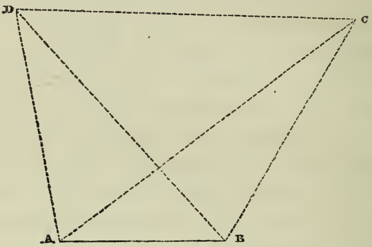
Hence the three required distances may be found by the following proportions; viz. Sine A F C : A C : : Sine F A C : F C; Sine A F C : A C : : F C A : F A; and lastly, Sine B F C : B C : : B C F : B F.

The results are $F C = 1042.66$, $F A = 709.33$, and $F B = 934$ yards.

N.B. If C had been nearer to F than A B, the general principles of construction and computation would be the same; but the angles equal to the observed ones, instead of being projected on the same side as C, should be set off on the contrary side of A B.

Example X.

Let the distance be required between two trees, as D, C, which are so situated that no nearer access to them is to be obtained, than at A and B. Having measured the station line A B = 113 perches, let the angles be taken, viz. D A B 100°, = C A B 36° 30′, C B A 121°, and D B A 49°.



180° — (D B A + D A B) = A D B = 31°.
180° — (C A B + C B A) = A C B = 22° 30′.

Therefore in the triangle A B D, to find D B, say,

Sine A D B : A B : : Sine D A B : D B
31° : 113 : : 100° : 216.07

And in the triangle A B C, to find B C, say,

Sine A C B : A B : : sine C A B : B C
22° 30 : 113 : : 36° 30′ : 175.64

In the triangle D B C, we have the angle D B C = (A B C—A B D) 72°, together with the sides B D and B C, as already found; therefore, to find D C, say,

B D + B C : B D—B C : : tangent $\frac{1}{2}$ D C B + C D B :
391.71 : 40.43 : : 54° : 8° 05′
Therefore 54° + 8° 05′ = 62° 05′ = D C B.
54 — 8° 05′ = 45° 55′ = C D B; and
Sine C D B : B C : : Sine D B C : D C
45° 55′ : 175.64 : : 72° : 232.49

The student who is desirous of pursuing this department of trigonometry, is referred to the treatises of Gregory, Bonnycastle, and Woodhouse.

HEIGHTS AND DISTANCES BY APPROXIMATE MECHANICAL METHODS.

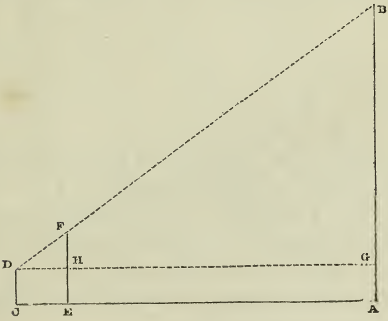
By Shadows.

Let a staff of given length be set up vertically; and when the sun shines, mark the extremities of the shadows of the staff and of the object at the same instant of time on the same plane; let the lengths of these shadows be respectively measured. Then suppose the summits of the staff and of the object to be connected with the extremities of their respective shadows, and by reason of the similarity of the triangles so formed, the proportion will be, As the length of the shadow of the staff, is to the length of the staff itself, so is the length of the shadow of the object, to the height of the object. Thus, let the length of the staff, above ground, be 10 feet; length of its shadow 15 feet; and the length of the shadow of the object 135 feet.

Then, 15 : 10 : : 135 : 90 feet = height required.

By Two Rods, or Staves.

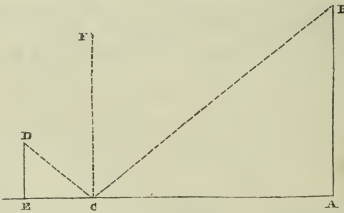
Let a perfectly horizontal plane be formed with planks or otherwise, on the same level with, or at a given height



above or below, the bottom of the object; and on such plane set up, vertically, two rods, one say, of 5 feet, the other of 10 feet in length. Let A B be the object, as a spire or tower, whose altitude is required, and A C the horizontal plane, whose height relative to the base is ascertained. Let the rods, C D and E F, be placed in a line with the object; and let one of them be moved nearer to, or farther from the other, until the summit, B, of the object, is seen in the same line as D and F, the tops of the rods. Then (Euc. El. B. 6, prop. iv.) it will be as D H (= C E) : F H : : D G (= C A) : B G ; to which add A G, that is, the height of the top of the shorter staff above the base, for the full height A B.

By Reflection.

Place a vessel of water, or other liquid, upon the ground, and recede from it, until you see the top of the object reflected from the still surface of the liquid. Then, it is well known, that the angles of incidence and reflection, viz., B C F and D C F, and consequently their supplements, B C A and D C E, are equal; we have also got two equiangular triangles; it will be, therefore, as C E is to E D, so is C A to A B; that is, as the horizontal distance from the point of reflection to the observer's foot, is to the height of his eye above the reflecting surface, so is the distance from the point of reflection to the foot of the object, to its altitude above the said surface.



PORTABLE BAROMETER.

As this instrument is inapplicable to the ordinary purposes of the surveyor, a description of it and of its application is omitted; the rather as ample instructions are given by several authors, Mr. Simms, already mentioned, among the rest. It is very properly called a "Mountain Barometer," being adapted for taking the relative heights of hills, where approximations only are required. It is expensive, and easily injured, and requires much accuracy of observation, one-tenth of an inch of mercury measuring 84 feet* of air, or of altitude.

HYDRO-PNEUMATIC BAROSCOPE.

A description of this instrument for determining heights, invented by Mr. Cooper, the eminent chemist, appears in the Philosophical Transactions, for 1839; from which the following extract has been made.

"The principle on which its construction is founded, is, that the volume of a given quantity of air under a constant temperature, is inversely as the pressure to which it is subjected; and the means employed to estimate the change of volume which that quantity of air undergoes, by being subjected to differences of pressure caused by a change of elevation, are the determination of the difference of weight which a floating body (containing air,) is capable of sustaining in both situations. Thus, if a vessel containing a quantity of air and water be floated in water, and there be a communication between the water in the floating body and that in which it floats, it will follow, that when such an apparatus is subjected to diminished pressure, the air within the float will dilate, and cause a volume of water equal in amount to the dilatation of the air to be driven out of the

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* Even this is by no means agreed upon; some authors stating 84, some 91, and others as many as 103 feet.

the float; and the difference of weight, which the floating body will sustain, will be the exact weight of the water expelled; if such an apparatus is subjected to an increased pressure, the air within it will contract, and consequently a quantity of water, (from that in which it floats) will enter the float, and the diminished weight it is capable of sustaining will be the weight of the water which has entered the float, in consequence of the diminution of the volume of the air." For a description, &c. of the apparatus, with illustrative plates, the reader is referred to the Philosophical Transactions.

PORTABLE BAROMETRICAL INSTRUMENT.

Sir John Robison, Secretary to the Royal Society of Edinburgh, has proposed a substitute for the Mountain Barometer. He says, " The instrument is a glass tube about 0.25 of an inch in diameter, and about 14 inches long, with a small bulb like that of a thermometer blown on the upper end. The stem of the tube has been graduated by divisions made experimentally by the instrument-maker* in the following manner. On a day when the barometer stood at 30 inches, and the temperature of the air was 62°, it was placed in the receiver of an air-pump, and when the rarefaction allowed the barometer of the pump to fall to 29 inches, the instrument was lowered until the open end of the tube became immersed in a cup of water, over which it had been suspended by a wire passing through a stuffing-box. On allowing the atmosphere to enter the receiver, the water was pressed up the tube until the density of the air corresponding to the state of the external atmosphere was restored, and the height of the fluid was carefully marked.† The instrument was a second time suspended in the air-pump receiver, and the exhaustion repeated until the barometer gauge indicated 28 inches; the immersion in the cup having been made as before the air rushed in, and the graduation of the tube corresponding to 28 inches was accomplished. By continuing this process, the graduation of the stem was carried as far as was thought requisite, (the intermediate divisions having been made in a similar, or more readily in a mechanical manner,) when the instrument became ready for use.

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* Adie and Son, Princes Street, Edinburgh.

† This is obviously a simpler and more practical mode of applying the principle adopted in the " Pneumatic Baroscope."

" It is obvious, that in this manner a traveller arriving at a station in the midst of mountains, and having with him a number of such tubes, would only require to send messengers to the summits of the hills, with one or more of these tubes, and a tin case containing water, for the purpose of giving him the means of determining their heights with considerable accuracy. Each messenger carrying with him the empty glass tubes, is to be instructed to insert their open ends in his flask of water when he shall have reached the summit, and to bring them down again." [Having, before immersion, exposed the tubes so that both they and the air within them shall have acquired the temperature and the tenuity of the surrounding atmosphere. The tubes from the time of their immersion until their examination, must, of course, remain in the water.] " Having done so, the air in the bulb and tube having thus become rarified to the tension of that on the top of the mountain, is compressed by the water which the increased pressure of the atmosphere, as he descends, forces into the tube; so that when he returns to the datum barometer, the water in the tube will indicate the height corresponding to the dilatation of the air at the summit of the elevation It is evident that in this, as in all barometical observations, the standard barometer should be read at the time of the assistant's observation; that is, simultaneously with the immersion of the tubes. To effect this, the usual arrangement is, to read the standard barometer at stated periods, say every half hour. If the temperature and degree of moisture of the air, on the mountain, and at the lower station, were alike, no further correction would be requisite; but as in the case of the barometer, so with this instrument, for minute accuracy a thermometer and hygrometer should accompany it, and be simultaneously observed, so as to permit the application of the usual corrections."*

"In many cases precisely equal temperatures may be obtained at the upper and lower stations by keeping the tin case supplied with water of melting snow, or ice."

"In a general and rapid survey of a country, such instruments would possess value from their portability and cheapness."

BY MEANS OF BOILING WATER.

Lieut.-Colonel Sykes, R.E., has lately applied the well-known principle that water will boil at different temperatures, according to its altitude above the level of the sea, to the measuring of heights; and as the apparatus is cheap, simple, easily made, easily used, and not so liable to injury as the mountain barometer, the following description and tables applicable to it, are extracted from the Geographical Journal.

A is a common tin pot, 9 inches high, and two inches diameter; B, a tin tube, made to slide up and down in the pot; the head of the tube is closed, but has a slit in it, C, to admit the thermometer passing through a collar of cork, which shuts up the slit when the thermometer is placed; D, thermometer, with so much of the scale left out as may be desirable; E, holes for the escape of steam.

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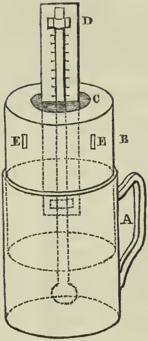
* This description has been taken from the "Transactions of the British Association" for 1839, politely corrected, by letter, by Sir J. Robison.



The pot is filled four or five inches in depth with pure water; the thermometer fitted into the aperture in the top of the sliding tube; and the tube pushed up or down to admit the bulb of the thermometer, to within about two inches of the bottom of the pot.*

Before using a thermometer for this purpose, its "boiling-point" for the datum level should be carefully ascertained by experiment, and the difference between that and 212° noted as an index error. The temperature of the atmosphere at the time of observation, should be noted.

To find the barometric pressure and elevation, corresponding to any observed temperature of boiling water between 214° and 180°.

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* This apparatus can be procured, ready for use, at Newman's, optician, Regent Street.

TABLE I.

Boiling point of Water.	Barometer modified from Tredgold's formula.	Logarithmic Differences, or Fathoms.	Total Altitude from 30 inches or the level of the sea.	Value of each degree in feet of Altitude.	Proportional part for one tenth of a Degree.
	In.		Feet.	Feet.	Feet.
214	31.19	0084.3	—1013	—505	..
213	30.59	84.5	507	507	..
212	30.00	84.9	0	+509	..
211	29.42	85.2	+509	511	51
210	28.85	85.5	1021	513	..
209	28.29	85.8	1534	515	..
208	27.73	86.2	2049	517	52
207	27.18	86.6	2566	519	..
206	26.64	87.1	3085	522	..
205	26.11	87.5	3607	524	..
204	25.59	87.8	4131	526	..
203	25.08	88.1	4657	528	53
202	24.58	88.5	5185	531	..
201	24.08	88.9	5716	533	..
200	23.59	89.3	6250	536	..
199	23.11	89.7	6786	538	54
198	22.64	90.1	7324	541	..
197	22.17	90.5	7864	543	..
196	21.71	91.0	8407	546	..
195	21.26	91.4	8953	548	55
194	20.82	91.8	9502	551	..
193	20.39	92.2	10053	553	..
192	19.96	92.6	10606	556	..
191	19.54	93.0	11161	558	56
190	19.13	93.4	11719	560	..
189	18.72	93.8	12280	563	..
188	18.32	94.2	12843	565	..
187	17.93	94.8	13408	569	57
186	17.54	95.3	13977	572	..
185	17.16	95.9	14548	575	..
184	16.79	96.4	15124	578	58
183	16.42	96.9	15702	581	..
182	16.06	97.4	16284	584	..
181	15.70	97.9	16868	587	59
180	15.35		17455		

TABLE II.

Multipliers to correct the approximate height for the temperature of the air.

Temperature of the Air	Multiplier.	Temperature of the Air.	Multiplier.	Temperature of the Air.	Multiplier.
32°	1.000	52°	1.042	72°	1.083
33	1.002	53	1.044	73	1.085
34	1.004	54	1.046	74	1.087
35	1.006	55	1.048	75	1.089
36	1.008	56	1.050	76	1.091
37	1.010	57	1.052	77	1.094
38	1.012	58	1.054	78	1.096
39	1.015	59	1.056	79	1.098
40	1.017	60	1.058	80	1.100
41	1.019	61	1.060	81	1.102
42	1.021	62	1.062	82	1.104
43	1.023	63	1.064	83	1.106
44	1.025	64	1.066	84	1.108
45	1.027	65	1.069	85	1.110
46	1.029	66	1.071	86	1.112
47	1.031	67	1.073	87	1.114
48	1.033	68	1.075	88	1.116
49	1.035	69	1.077	89	1.118
50	1.037	70	1.079	90	1.121
51	1.039	71	1.081	91	1.123

When the thermometer has been boiled at the foot, and at the summit of a mountain, the number in the fourth column of Table I, opposite the boiling point below, is to be deducted from the number in the same column opposite the summit boiling point; this gives an approximate difference of height between the two stations, which is to be multiplied by the number opposite the mean temperature of the air in Table II, for the correct difference of altitude. The operation is performed in a few minutes, the observer being provided with a spirit lamp, and some lucifer matches, or other convenient modes of obtaining light and heat.

Example I.

Boiling point at summit of Hill Fort of Púrundhur, near Puna 204°.2		204°	4131	
Deduct proportional part		.2	104	4027
Boiling point at Hay Cottage, Puna 208°.7		208°	2049	
Deduct proportional part		0.7	357	1692
				2335
Temperature of atmosphere at summit	75°			
Ditto, below	83			
Mean	79 = Multiplier			1.098
Correct difference of altitude				2564

When the boiling point at the upper station only is observed, and for the lower, the assumed height at the level of the sea (that is 30 inches), is taken, then the observed barometric reading may be converted into feet, by subtracting the logarithm of its height, in inches and tenths, from the logarithm of 30 inches (log. 1.4771213); in their difference setting off four figures from the left, for integers, and multiplying this same difference, so pointed off, by 6*; as the differences in the second, or barometer column of the table, vary more rapidly than those in the fourth, or feet, column.

---

* Because the difference of the logs., so pointed off, will be the approximate height in fathoms. "Hutton's Mathematics," and other works.

Example II.

feet.
Boiling point at upper station . 185° = 14548
inches
Barometer at Calcutta (at 32°) 29.75
Log. diff. = 1.4771213 — 1.4734870 = 0036.343 × 6 = 218

Approximate height 14330

mean
Temperature at upper station 76° } 80° = multiplier 1.100
Ditto at lower . . 84° }

Correct altitude 15763.000

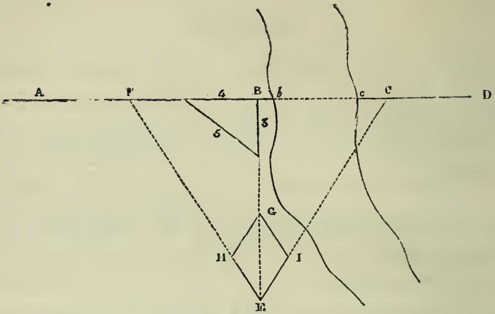
It will be perceived, that in Table I, it is assumed that water boils at 212°, the barometer standing at 30 inches at the level of the sea, which is taken as zero; if this were true, and that at a given station, water boiled exactly at any full degree of temperature, the altitude would be obtained by inspection in the fourth column of Table I, merely correcting for temperature of the air traversed, by Table II; but the level of the sea is as yet undefined, and thermometers are not always accurately divided; these points must, therefore, be verified by experiment, as already stated.

DISTANCES.

Example I.

The measurement of the line A D, is obstructed by a river; to determine the width of the river.

At E set up a mark at right angles to A D from the point B, and about the width of the river, or other obstacle. If a theodolite is used, it being adjusted at E, let the vernier plate be clamped at zero or any other marked division, and set on B; let the lower plate then be clamped, and the vernier plate released, turn the telescope on a



mark, C, at the other side of the river, and again clamp the upper plate ; now unclamp the whole head of the instrument, and turn the telescope towards B, clamp the lower plate, and releasing the upper one, turn it until the vernier indicates zero, or whatever degree it was first adjusted to, and the point F, in the line A D, which is intersected by the cross hairs, will be at the same distance from B, as C is from B ; because the triangles E B F and E B C are similar and equal. (Euc. El. B. i. prop. xxvi.) It will be observed that in this case, having set out the right angle at B, no further reading of angles is necessary. The points B and C may be taken at any convenient distances from the water's edge, and the distances *B b* and *C c* being measured and deducted from B C, the remainder will of course give the width of water-way. This might obviously be accomplished with any other instrument for measuring angles, observing merely to make the angles B E C and B E F equal to each other.

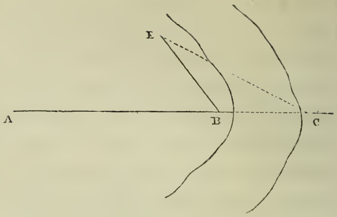
Or with the Chain.

It will be necessary first to explain how a right angle is set out with the chain only. The square of 5 being 25, equals the sum of the squares of 3 and 4; that is $25=9+16$; therefore if 3 and 4 be assumed as the length of the legs of a right-angled triangle, 5 will be the length of the hypothenuse, consequently, if three portions of the chain bearing this proportion to each other be taken, a right angle may be set out with them. The greater the length of the sides is, the less considerable will the errors be. If 3, 4, and 5 be multiplied by 8, the products will be 24, 32 and 40, the sum of which, being 96, gives the longest parts into which the chain is divisible, but any other multiples of the first figures may be used, as 6, 8, 10, or 12, 16, 20, as may best suit the ground.

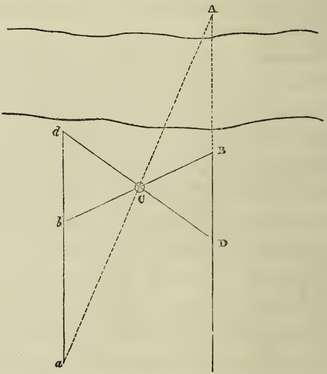
Referring to the last figure, having set out the right angle at B, take E B of any convenient length, and holding one end of the chain at E, let the centre of it be drawn to I, in a line with C; and holding the chain fast atE and I, let the remaining end be drawn tight to G, in the line E B; the ends of the chain being now held fast at E and G, and the point I marked, let the centre of it be moved to H, when we shall have the triangles B E F and B E C, equal, as before, and consequently B C = B F. The accuracy of this method will necessarily depend upon the correctness of the chain, and the care taken in the several operations. It is manifestly not necessary to find the points H and I by the centre of the chain, but it is desirable that the sides of the triangle be taken as long as the ground will admit of.

ANOTHER MODE.

If it be inconvenient to measure a right angle at B, lay out any other angle; measure B E, and make the angle B E C equal to half A B E; then will B C = B E; for by by Euc. El. B. i. prop. 16, the angles at E and C are equal; and consequently the sides B E and B C opposite the equal angles are equal. It may be observed however that a right angle can almost always be measured at one side or the other of a line, at one side or the other of the river.



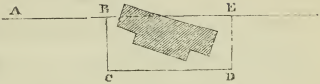
Or,



Take any distance D B, say one chain, and on it construct any triangle, as D C B, produce the sides, making C *d* = C D, and C *b* = C B; now produce the line *d b* to *a*, that is, until *a*, C, and A, are in one right line; then will *b a* = B A; for the triangles D B C and *d b* C, being mutually equilateral, are equiangular; therefore the angles at D and *d* are equal; but D C A and *d* C *a* are opposite and consequently equal angles; the triangles D C A and *d* C *a* are therefore mutually equiangular; consequently C D is to D A, as C *d* to *d a*; but C D and C *d* are equal, therefore D A = *d a*; and deducting the equal lines D B and *d b*, from those equal lines, the remainders, B A and *b a* are manifestly equal.

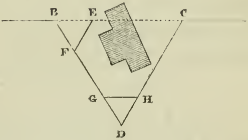
Note. It is obvious that the distance from a given point to any inaccessible object, may be as easily obtained by these means, as the width of a river.

Example II.



To pass a house, or other obstruction, in a line A B. Turn off at right angles, as at B, measure B C; at C set out another right angle; measure C D, and again, at D, turn off at right angles; make D E = B C, which will evidently bring you into the original line, having C D equal to the distance B E; the necessary offsets being of course taken on C D.

Or,



Having measured to B, set out an angle of 60°; measure B D, and at D set out another angle of 60°, the triangle B C D will then of course be equiangular, and consequently equilateral, that is B C will be equal to B D or D C, therefore the measurement of the main line may be resumed.

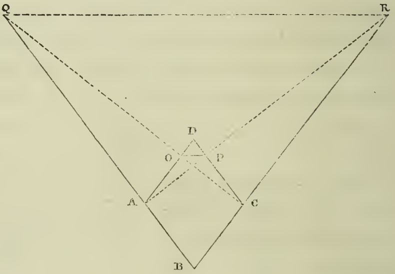
To accomplish this with the chain only. Mark the point B, and measure forward 50 links, in the same line, to E, then holding the ends fast at E and B, let the centre be drawn tight to F, measure in the line B F any convenient length, as B D; from D towards B take 50 links to G, let the ends of the chain be held at D and G, and the centre drawn to H; in the line D H measure a distance equal to D B; and B C will be equal to B D or D C as before, and for the same reason.

It is evident that if the obstruction were a river, pond, &c. which did not interfere with the sight of the line, the distance H C need not be measured.

Example III.

To find the length of an inaccessible line, as Q R.

At any convenient point, as B, lay down the rhombus, B A D C, so that two of its sides, B A, B C, are directed to the extremities of the line Q R. Mark the intersections O and P of A R and C Q with the sides of the Rhombus; then the triangle O D P will be similar to the triangle R B Q.



For as A D and B R are parallel, the alternate angles D A R and A R B are equal; also the angles A D P and A B R being the opposite angles in the parallelogram, are equal. Hence P D : D A :: A B : B R; therefore D P × B R = D A × A B. In the same manner it may be shown that D O × B Q = D C × C B. Now since D C equals the opposite side, A B, and D A equals B C, we have D C × C B = D A × A B, and therefore P D × B R = D O × B Q. Hence P D : D O :: Q B : B R; and, as the contained angles are equal, the triangles O D P and Q B R are similar. Q. E. D.

Then as all the sides of the rhombus are equal, the inaccessible distance R Q will be equal to $\frac{OP \times BA_2}{OD \times DP}$

For $PD : DA :: AB : BR = \frac{AB^2}{PD}$

and $OD : OP :: BR : RQ = \frac{AB^2 . OP}{OD . DP}$

Thus if $BA = BC$ &c. $= 50$ links, $OD = 8$ links, $DP = 10$, and $OP = 12$ links, then $QR = \frac{2500 \times 12}{8 \times 10} = 375$ links.

MICROMETER.

A micrometer, which is adapted for measuring very small angles, is sometimes attached to the telescopes of theodolites or levels, and is applied to the determination of distances, by measuring the angle subtended by an object of known dimensions, placed at the remote extremity of a line whose length is required. A man, a door-way, &c. will give an approximation. But the vertical hairs of a level may be conveniently applied to obtain approximate distances, when those are sufficient for the purpose.

TO MEASURE INACCESSIBLE DISTANCES BY THE SPIRIT-LEVEL.

Mr. Bunt, of Bristol, has suggested,[*] that the vertical hairs of a level may be applied to the measurement of distances, in the following manner. Let a scale, properly divided, be held horizontally, at any distance in front of a level: and the angle subtended, that is, the portion of the scale included between the vertical hairs, will be in proportion to the distance of the scale from the telescope. To reduce this to practice, let a given distance, as one chain,

[*] See Transactions of the British Association for 1838.

be measured from the instrument; and at that distance note what portion of the scale is included between the hairs; which may be taken as unity. Then the portion of the scale included between the hairs at any other distance, will be to that distance, exactly in proportion as the quantity found at the initial point, was to the initial distance. Thus suppose at one chain, two tenths of a foot, were included; at two chains, four tenths would be the reading; at two chains 50 links, the reading would be five tenths, or half a foot, and so on; each hundreth part of a foot representing five links. It will be found that an approximation only, can be obtained by this method; for it is not possible, under ordinary circumstances, to read the scale with sufficient accuracy for anything more.

BY THE VELOCITY OF SOUND.

Let a gun be fired at the remote extremity of the required distance, and observe by means of a Chronometer, or Watch, adapted for giving parts of seconds, the interval which elapses between the flash and the report; then estimate the distance for one second by the following rule, and the product multiplied by the observed interval of time, will equal the distance required.

At 33° of Fahr. the velocity of sound is 1100 feet per second.

For lower temperatures deduct half a foot for every degree below 33°.

For higher temperatures add half a foot for every degree above 33°.

Thus at the temperature of 60°. the velocity of sound in air* is $1100 + \frac{1}{2}\,(60° - 33°) = 1113\frac{1}{2}$ feet per second.

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* It has been ascertained by experiments on the lake of Geneva, that the velocity of sound through water is four times greater than through air.—"Annales de Chimie," 1829.

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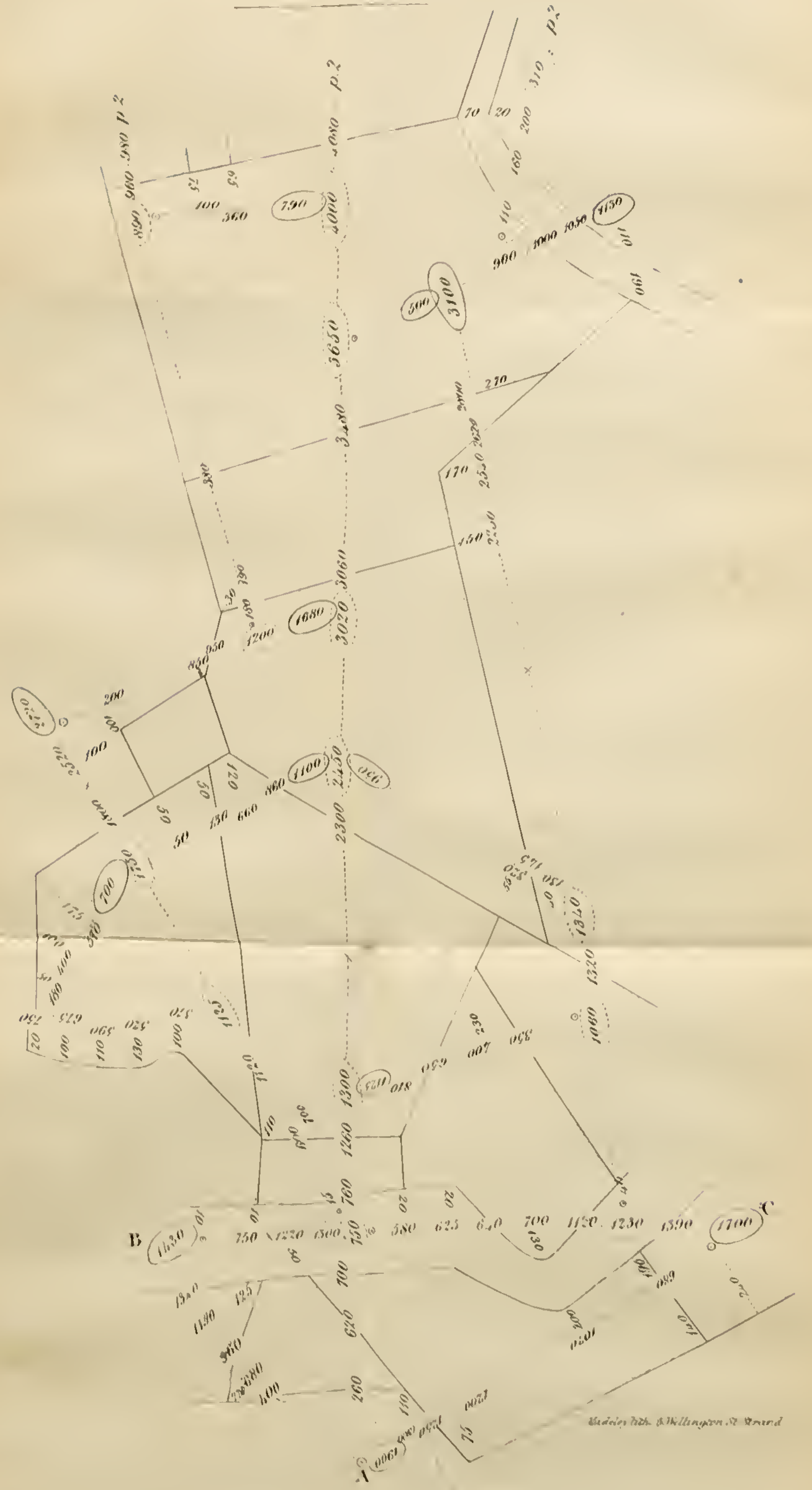

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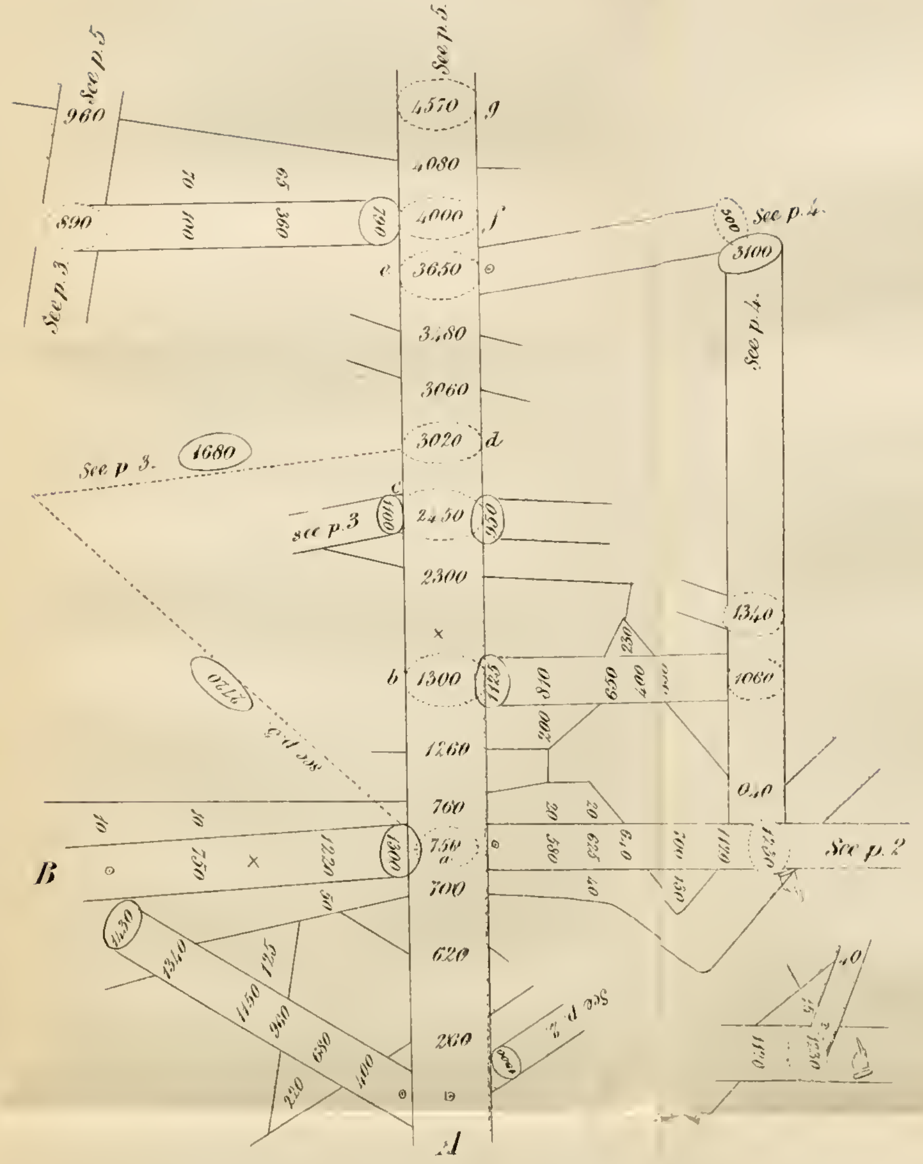

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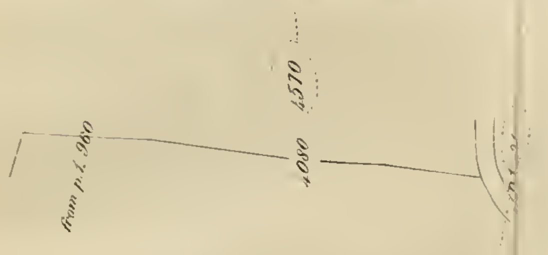

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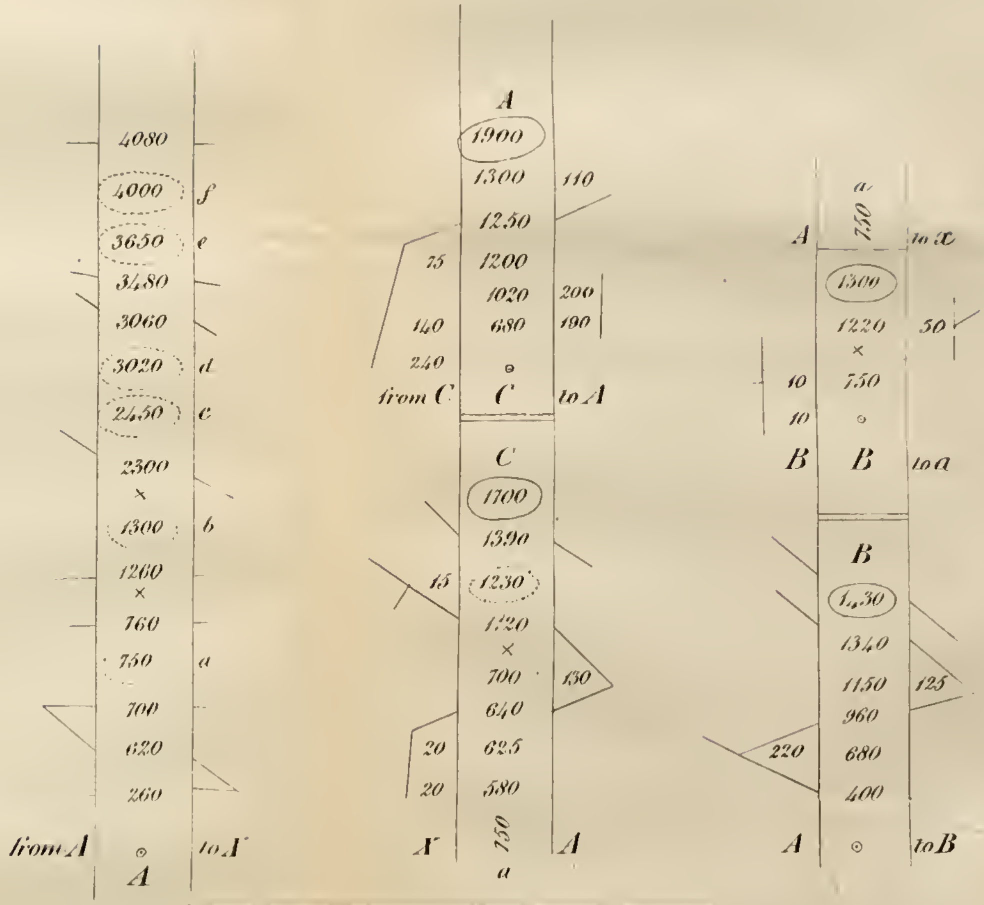

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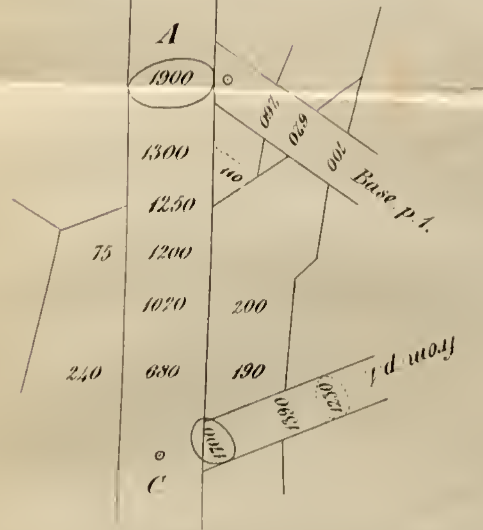

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PRACTICE OF LAND SURVEYIN
Section 4.

THE FIELD BOOK.

As there is some difference of opinion as to the best mode of keeping a field-book, the methods commonly in use are given. The first, or sketch field book, is that generally adopted by the best surveyors in the kingdom. Many surveyors write their distances horizontally, if the expression may be permitted, that is, in the same lines as the offsets, as from A to B, field-book No. 1; but it will be found much less liable to error to write them in the direction of the station line, and the offsets at right angles to them; as in the remainder of the same figure.

A small × should be made at every change of arrows.

The second method is that used on the Ordnance survey of Ireland.* In this case, as the distances are written between two ruled lines, and the offsets outside, the two series of figures do not interfere with each other.

It will be observed that the lines are sketched in the directions in which they run; and the distances written in the direction in which they were measured. Much time may frequently be saved by observing the order in which lines ought to be taken. Thus, in the first example, if 750 of the base be measured, poles being left at ⊙ and at 750, and then the line to the right measured to 17 chains, leaving a pole at 1230; the 19 chains back to ⊙ should then be taken; after that the line A B; and from thence back to 750 on the base. By this arrangement the work may be done by going over the ground once only; whereas by slovenliness, or inattention, a surveyor might easily contrive to make it appear to be necessary to go over the ground three or four times, for the same purpose. The secondary, or false, stations, are surrounded by dotted lines,

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* The Author was employed for a considerable time on that important survey.

(and this should be rigidly observed), to distinguish them from the principal ones, or those at the extremities of the lines. Where secondary lines spring from others, the station distances of the principal lines should be written in the direction in which they were measured, with regard to that secondary, as at C, Field-book No. 2; and the references to the secondaries should be written, in position, on the main line, as at *d e*, Book 2. These references should be written in the field, as they occur, and nothing left dependent on memory afterwards. This cannot be too strongly impressed on the learner. The very best memory will be found to be exceedingly fallacious; the mind of a surveyor in the field is engaged on so many matters, and minute occurrences are frequently so similar, that to remember one in particular is next to impossible. Let him then keep his field-book in such a manner that another person may plot it as readily as he could himself. Some surveyors keep a particularly neat field-book, and spend much time in making accurate sketches; that is a matter of taste; but the time spent in securing distinctness is never thrown away. If any confusion in the notes should occur, it is advisable to make a separate and distinct note of the confused portion, as at C, fig. 2.

In extensive surveys the base lines should be distinguished by letters, that so the field-book may be indexed, to facilitate reference. Thus, in the example, Field-book No. 1, say the first line of the base is called A X; in the index it would be A X ⊙ to 4000, p. 1.

〃 4000 to 1 mile 1000, p. 2, &c. Then the references to the subsidiary lines being duly entered in their places, in those pages of the field-book, any portion of the work may readily be found. The surveyor will find it advisable to avoid making any two lines in the same vicinity, of exactly the same length.

The third method of keeping a book is much practised by old surveyors.

It is well to date a book every day, but the dates of the commencement and termination of a survey should never be omitted.

It is generally directed, in books, that field notes be kept in ink; but many of the most eminent surveyors in the kingdom keep theirs in pencil, which is by so very much the more convenient way, that unless in government, or public surveys, where the books are given in and kept as registers, it is decidedly to be preferred. It is desirable that none but a good drawing pencil, of medium hardness, be used, and that kept in proper order.

When it is necessary to return field-notes in ink, they should be taken in ink; as, independent of the irksomeness of re-writing, it destroys the identity and originality of the notes, and if a mis-reading should occur, no means of rectification remain, unless indeed the whole be copied into another book.

The angle, or bearing, when an instrument is used, should be written distinctly in such a manner as not to admit of its being mistaken for any other number. It is done differently by different surveyors. Some write the angle at one side of the line, in the same direction as the distances of that line, but with a strong mark under the angle, as in the annexed fig. 1. Others write in the same column as the distances, as in fig. 2; whilst others sketch the arc and write the angle, as in fig. 3. Either way will answer the purpose; but the student should adopt one of them, and abide by it, as uniformity of practice is one of the best securities against error.

FIG. 1.

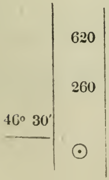


FIG. 2.

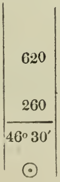
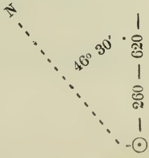


FIG. 3.



OFFSETS.

Offsets ought not to exceed about one chain in length, in content surveys; though in Parliamentary surveys a greater latitude is allowable. In this a surveyor must exercise his discretion. Important points, as houses, canal banks, &c. should be well fixed, but minute accuracy in field fences is not always so necessary in Parliamentary work.

Offsets should be taken to the point intended to be plotted; thus in measuring to a brook, a drain, a wall, or a double fence, the centre of which is the boundary between two properties, the measurements should be made to that centre. In measuring a common single hedge and ditch, the general rule is that the outer edge or brow of the ditch is the boundary; sometimes however it is at the root of the quick. These local peculiarities should be carefully ascertained previous to the commencement of a survey. In measuring across fences these same observations of course apply. The width of a fence does not concern the surveyor; but the side to which the fence belongs should be shown in the Field-book, and on the plan, by a distinctive mark. Drains, rivulets, &c. which are to be represented by double lines, should be so sketched.

ACREABLE SURVEYS.

As the principles on which the surveys of parishes now (1842) in progress in England and Wales, for the commutation of tithes, are conducted, are applicable to all extensive surveys, the following remarks and instructions are amplified, adapted, and adopted from the report of Lieutenant Dawson, R. E., to the Tithe Commissioners.

To assist in the explanation three outline plans are appended. (Pl. I.) The object of the first two is to show how the particular lines, recommended to be measured, will ensure the accuracy of the general outline, the gross contents, and the relative position of the parts.

The third will point out their use in combining old maps of detached portions, in the formation of a new plan of the whole.*

DESCRIPTION OF PLAN.

The plan should represent accurately, and in their true relative positions, the several objects which now occupy the surface of the ground; such as Roads, Rivers, Lakes, Ponds, Canals, Streams, Drains, Parks, Woods, Fences, Houses and other Buildings, Ridings, Lathes, Wapentakes, Rapes, Hundreds, Parishes, Townships, &c. These may all be represented according to a Table of conventional signs, as represented in Plate II.

The boundaries or limits of all lands, and parcels of land, should be marked on the plans, whether they be defined by fences or not. Where no boundary fences appear, the limits should be shown by a dotted line.

Each separate parcel of land should be numbered upon a plan, the numbers following in succession from 1 to the highest number required. These numbers will refer to a schedule, or table, in which should be specified the name and description of each field or inclosure, with its true quantity or contents; the names and description of the several proprietors and occupiers thereof; the state of cultivation of the several lands, whether as arable, meadow, pasture, wood, coppice, or common-land, garden, orchard, hop-ground; or otherwise.

Roads, Rivers, Lakes, Ponds, and Canals, should be numbered upon the plans in the same manner as inclosures, and their contents given separately in the schedule.

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* Those who require more detailed examples of triangulation, are referred to "Practical Land Surveying," by Thomas Holiday, York.

The plans for the tithe survey are required to be drawn to a scale of three chains to one inch, to admit of the correct computation of the contents of the several lands. Lieut. Dawson observes " this is, in fact, the smallest scale which can be relied on for computing contents from the map itself, and is accordingly the scale upon which the great majority of existing maps in the country have been, in the first instance, constructed." Many good maps, however, have been constructed at four, and at five chains to an inch ; and the contents on the ordnance survey of Ireland are partly computed from the maps themselves, although laid down to a scale of 6 inches to the mile, or upwards of 13 chains to an inch. Four chains to an inch is a very good scale, because it admits of being readily reduced to a Parliamentary scale, and is of sufficient distinctness for most purposes.

It is usual to place the north of the plan towards the top ; writing the name of the estate, parish, &c. as a title, with that of the county in which it is situated ; and adding the name and address of the surveyor, the date of performance, the scale, and the total contents.

It is desirable to determine the area of the whole parish, estate, &c. by some means which make the correctness of that area independent of the result obtained by summing up the contents of each inclosure, minute errors in many of which would escape observation, if not checked by comparison with the correctly ascertained whole. It is essential, in fact, to arrive at the total area by direct admeasurement of the space included within its external boundary, and the simplest means for effecting this object will be found to be :—

1st. To measure two straight lines through the entire length and breadth of the estate, parish, &c.

2dly. To connect the ends of these lines by means of other measured lines; and

3dly. From these connecting lines, by measured triangles and offsets, to determine the entire boundary.

The true area may then be obtained by calculation from the measured distances, and by the admeasurement of the included space upon the plan.

The object and application of these lines will be better seen by reference to the accompanying plate No. 1.

Fig. 1. is a rough sketch of a parish (Eye) to be surveyed.

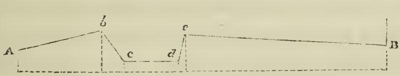
The two main lines recommended to be measured through it are marked A B and C D.

AC, CB, BD, DA, are the connecting lines.

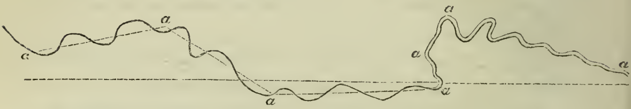
T T T T are the triangles constructed upon the connecting lines.

OFFSETS.

a a a a are the offsets, or perpendicular distances of the several angular points of the boundary from the measured lines; and here the learner may be reminded, that, if the two extremities of a straight line, as a fence or a portion of a fence, be fixed by offsets, no intermediate ones are necessary. Thus in surveying the fence A B, if



offsets are taken to A *b c d e* and B, they will be quite sufficient for all purposes, however long may be the intervals between them. All offsets must be taken at right angles to the lines from which they are measured. When the boundary consists of a very crooked brook it would be an interminable task to measure to all its tortuosities; balance lines of reasonable lengths may in such case be assumed, on the ground, and offsets taken to their extremities; observing to take such as will denote all remarkable features; and sketching in the minutiæ between them. Thus in the accompanying sketch, the brook might be delineated from offsets measured to the seven points marked *a*; whereas upwards of sixty would be required to enable a person to plot the several sinuosities minutely. When a river or considerable stream forms the boundary, it will generally be sufficient to take offsets to one of its banks, ascertain the width at several places, and sketch in the opposite bank.



Returning to fig. 1. plate I, if the main lines A B and C D be measured accurately, and the true lengths from the point C, at which they cross one another, be laid down on the plan, it will be seen that the connecting lines A C, C B, &c. will form an efficient check on the general direction of the two main lines, with reference to each other.

A satisfactory check on the lengths of the several lines will by the same means be afforded; for as the points A, C, B, D, are in each case determined by the intersections of three lines, an error in any one of these lines must immediately be evident.

Base or Main Lines.

Previous to his setting out his main lines a surveyor should walk over the lands he is about to survey, to ascertain the most advantageous directions in which to run his lines. Having decided upon one terminus, which should if possible be upon a fixed and conspicuous object, let him ascertain if there are any considerable obstructions, as woods, houses, quarries, ponds, &c. likely to come in the way; and if there are, go and ascertain the best mode of passing them, and set up a pole accordingly; thus fixing two points, and so deciding the direction of one line. If there are two or more considerable obstructions, the points for passing them should be marked by poles, and the direction of the line determined by means of them, as it is much better to work *from* fixed points than *to* them. The direction of the main transverse line should be decided upon the same principles.

A surveyor having ranged his main lines, as A B, C D; and the connecting lines, as A C, C B, B D, and D A; will find it most economical to measure each large triangle separately. Thus let the line A O be measured with due care; leaving false stations, or marks, in advantageous situations for the measurement of each fence; note the crossings of all fences, &c.; and mark the point O in a durable manner, by sinking a large stone, driving a substantial post in the ground, or otherwise. Measure from O to C in a similar manner; and also from C to A; and then proceed to fill in the interior of the triangle A O C. By this means much time and labour will be saved; as if the base lines be measured throughout in the first instance, many important particulars will escape the memory, marks will be obliterated, &c.; so that it would be found necessary to measure the base lines at least twice over.

In a content survey a triangle should never be measured without a check line. Thus in the triangle A E F, fig. 1, plate I, a line, as *a* F, should be run from the base, A F, to some point, as *a*, in the opposite side, A E; otherwise a serious error may exist without being discovered; for, it is evident that two lines of any length would form a triangle with the base A F, provided the sum of their lengths exceeded the length of the base A F; there is therefore no security for the correctness of either of those lines unless a check line be run. The bases or main lines being duly measured, every point in them may be treated as a fixed point; so that the accuracy of lines, as H N, I P, &c. run from one base to another, is ensured; and the points in those cross lines also become fixed. It will be observed that in case of a surveyor resolving to fill in each large triangle as he proceeds, a line, as H M, run across that triangle will satisfy him of its correctness. If an old map of the whole, or even part, of the property about to be surveyed, can be obtained, the lines should be laid down upon it, and thus a rough diagram be formed, on which the work may be advantageously projected, its progress registered, &c. If the work is laid down as it proceeds, errors will be the more readily discovered and corrected; in fact in an extensive survey, this cannot, with safety or propriety, be neglected.

In hilly districts much care should be taken to reduce lines measured over steep slopes to the horizontal plane. This may be effected in several ways, according to the importance of the work in hand. If there is much hill work to be done, or if extreme accuracy be necessary, a theodolite must be had recourse to.

If any offsets occur on the inclination, the deduction should be made at every chain, otherwise the offsets will be thrown out of position; but if there are no offsets, the deduction may more conveniently, and more accurately be made at the end of the inclination; it should however be done in the field, otherwise it will be exceedingly difficult to avoid errors.*

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* See Table (p. 13.) for the reduction of hypothenusal to horizontal distances, which may with advantage be copied into the field book.

If the length of an inclination is considerable, the work important, and no offsets to be taken, the base, or horizontal line, may be readily ascertained by right angled trigonometry, having the angle of elevation or depression, the hypothenuse, and the right angle at the base.

It is a very usual practice with surveyors, in measuring up or down a slope, to hold up the lower end of the chain until it comes level with the higher end, and then drop a stone from the chain-end, to denote where the arrow is to be placed; and for ordinary purposes this would come pretty near the truth, if done with care; but both ends of the chain should be held by assistants, and the height to which the lower end is required to be raised, be determined by the surveyor himself, standing at some little distance. On steep inclinations the chain must be taken in lengths, sometimes of 20 or 30 links each, and here very great care is requisite, to arrive at anything like the truth. To measure down such a hill, the whole chain should be stretched in line, the upper end held fast on the ground at the upper arrow, and a length of, say 30 links, held up to the level; a plumb line suspended, or a stone dropped, in the line, and the place marked; the back chainman should then come forward, and hold the thirty-link brass in the place marked, the leader go forward and proceed in a similar manner, to the end of the chain; observing always to stretch the chain in line first, and that the back chainman comes forward and holds down the same link as has been held up by the leader. To measure up a steep incline, it is necessary to observe the same precautions with regard to keeping the chain in line, and to hold the lower end of each length up to a strong pole, held plumb. If any of these precautions are neglected, errors will be the inevitable consequence; for the surface of the slope might as well be measured as a zigzag horizontal line; and if the hinder end of the chain be drawn forward at every length, one means of keeping in the line is lost, and it will be very difficult to avoid errors in summing up, more especially as such slopes are generally wooded; and frequently even one chain's length cannot be distinctly seen at once.*

As an author,† whose work has gone through several editions, has taken a good deal of pains to demonstrate, somewhat dogmatically, the propriety of giving the *superficial area*, and not the *horizontal plane* of a hill, it may be necessary to say that the latter is that universally intended to be given by practical surveyors; and this arises chiefly, if not altogether, from the impossibility of laying down the superficial areas of undulating surfaces, and of plane surfaces, on one and the same plane. It is evident if part of a line be level, and part measured over the surface of a hill, that if such a line be laid down at full length on paper, and the extremities connected by lines horizontal throughout, such lines will not close on paper, because the hill line will project beyond its due limits, hence a correct plan could not be constructed. Mr. Crocker calls those who advise the measurement of horizontal planes "advocates for a short measure," and says it is "an erroneous doctrine," but he has failed to show how superficial lines can be plotted; and it should be borne in mind that, as one practice is universal, no single person is aggrieved; and the value of land depends more upon local position, nature of the soil, &c. than upon acreable content.

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* When it can possibly be avoided, as by measuring to the extremities of such obstructions, main lines should never be measured over steep slopes. An instance occurred on the Irish Survey, within the Author's knowledge, where the length of a line passing over precipitous rocks had been well ascertained by calculation, but which it was desirable to measure on account of a boundary, was measured nine times over, and could not be correctly done at last.

† A. Crocker.

Besides, as both landlord and tenant are cognisant of the mode of measurement adopted, they make their bargain accordingly.

As it is exceedingly difficult to measure accurately over precipitous rocks, through woods, houses, ponds, &c. care should be taken to avoid running lines on such objects, but where that is unavoidable and yet the lines cannot actually be measured, recourse must be had to some of the modes already given for passing them.

Fig. 2, pl. I, is a rough sketch of a parish (Charlton) of which a portion is detached; any error in its position might prove a source of dispute with the intervening parish (Kidbrook) should the fences which now mark the boundary be at any time obliterated; the importance of the measured lines A B, B D, and A D, in determining with precision the true position of this portion of the parish, will therefore be apparent.

It will often be found necessary, not merely to test the accuracy of old maps, but to test and combine estate surveys of different portions of a parish or district.

In fig. 3, which is a more finished outline of the same parish (Charlton), the portions, coloured green, may be supposed to represent estates, of which plans have already been made; and we may suppose it to be required to combine these estate plans, in forming a new plan of the entire.

By reference to the preceding figure, 2, it will be seen that the positions of several angular points (such as X X X) in the boundaries of the estates, have already been ascertained by the survey of the main lines. These points may therefore be laid down at once upon the large plan, and the whole of the estate plans may, by means of these guiding points, be introduced bodily in their true places, leaving a comparatively small portion of the remaining detail of the parish to be supplied.

FIXED OR NATURAL OBJECTS.

In fig. 3, are shown a number of lines drawn from Charlton church to several conspicuous objects in adjacent parishes, which Lieutenant Dawson states, appear to him may be advantageously selected for the purpose of completing the interior detail of the parish. When such a proceeding becomes necessary, the direction of such lines must be determined by setting up a theodolite on the roof of the church or tower, and ascertaining the point over which the theodolite stands. If the outer wall be perpendicular the distance from it to the theodolite may of course be measured, and the point below determined by dropping a bullet or other compact heavy substance; but if this cannot be accomplished, it will be found difficult to fix the point. A very pretty theoretical mode of correction for this eccentricity, termed the "Reduction to the centre," is given by Lieutenant Frome,* to which the reader is referred; but, as in this case, the several distant stations could not be observed from below, its general applicability to practical surveying is questionable; besides, the raising of an instrument to such a position is troublesome, dangerous, and expensive. Here the student may be warned, that much care is required in measuring towards a distant fixed object, as a remarkable looking tree, a chimney, a windmill, or even a spire. Distance changes the appearance of objects, and their adjuncts, so materially, that a little inadvertence may lead to serious inconvenience or errors. The moment the appearance of an object, on which a surveyor is chaining, begins to change, or to be indistinct, the line should be marked by poles. Fixed objects should not be measured up to, as stations, on account of the difficulty already mentioned of determining the angular points; neither should they be approached too nearly, lest they obstruct the view of other stations; their positions may be determined by intersections, by running a line or lines to, or past them, &c. Therefore, although the selection of distant natural objects to chain on, will frequently save much time, in walking over the ground, yet they furnish abundant exercise for the surveyor's discretion.

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* Outline of a Trigonometrical Survey, &c.

Boundaries.

On the plan of a parish, or other public territorial division, the boundary should be shown by a dotted line. When it passes along the middle of a fence, the dots should be drawn on both sides.

When a road or stream forms part of the boundary, both fences should be shown, and the dotted line in the centre. The abutting of all external fences on a boundary should be shown. This observation applies to rivers, drains, droves, &c.

When a boundary passes through a field or other inclosure, without being defined by a fence, the whole of such field or inclosure should be shown on the plan, with the boundary, marked by a dotted line, passing through it.

In all cases of fences, the actual boundary line of the adjacent properties should be laid down upon the plan; and when the fence belongs entirely to one property or the other, that should be indicated by a distinct mark.

It is evident, that if a survey be conducted in the mechanical manner here described, no angular instrument will be necessary, as the position of every line is determined by its length. Of course every line must be measured exactly straight from point to point.

All triangles should be as nearly equilateral as possible, so that one line shall not run very obliquely into another; because, when two lines meet obliquely on paper, their point of intersection cannot be accurately ascertained.

When the direction of a long line has been determined, it will be found very advisable to set up a theodolite at one extremity, and, having set the cross hairs on the object at the other extremity, to clamp firmly the horizontal plates; and by means of the cross hairs, and vertical arc, to mark points at convenient distances in the line. This will in fact be indispensable in a hilly country. If the line be of considerable length, the instrument should be set up at other points in the line,* that so the straightness of the whole may be insured. In measuring over a precipitous, or steep hill, if the instrument be set up advantageously and accurately in the line, on the brow, the horizontal limb carefully levelled, the zero of one vernier set at 360°, the cross hairs on the back object, and the lower limb properly clamped, the telescope will of course coincide with the line; then, either unclamp the upper, or vernier plate, and turn it round until the zero of the other vernier comes to 360°; or what is better, when the ground is not too steep, is having the clips open, and the telescope coinciding with the line, reverse it in its ys, that is, turn it carefully, end for end; any error of reading the horizontal arc will be thus avoided. By this means it is evident that points may be marked on the hill, in the line. It is scarcely necessary to add, that a line may be marked down a hill, in a similar manner. Great care is requisite to keep a good line over hills, especially in a close country; in a content or acreable survey, angles, or bearings, ought not to be depended on; but if proper foresight and circumspection be used, all ordinary difficulties may be directly overcome. The mode of measuring over undulating ground, has been already described.

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* A ready mode of finding a point in a line, approximately is, when you think you are nearly in it, to lay a stick on the ground, with one end in the direction of one extremity of the line, then turning in the contrary direction observe if the other end of the stick points to the other end of the line; if not, move the stick to the right or left, as may be necessary.

AREAS OF PLANE FIGURES.

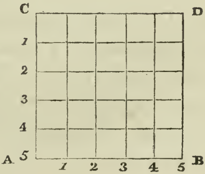
The area of a figure, means the measure of its surface, or the space inclosed within the boundaries of that surface, estimated by the squares of some known lineal measure, as inches, feet, links, chains, &c.; which in land measuring are converted into square perches, roods, and acres, &c.

The statute acre is equal to 10 square four-pole chains; that is, 10 chains in length, by 1 in width. Therefore the area in square chains and decimals, is reduced to acres and decimals by dividing by 10; that is, removing the decimal point one figure to the left, thus 100 square chains are equal to 10.0 acres

PROBLEM I.

To find the area of a square.

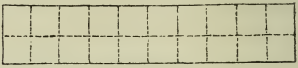
The square of one side, as A B, will be the area. Or call A B the base, and A C the perpendicular; multiply one by the other, the product will be the area.



This is evident from the accompanying figure, for if A B be divided into any number of equal parts, as 5, and

A C into an equal number, it is manifest that the figure will contain 25 of such parts, that is 5^2, or 5×5.

Hence the area of any *rectangular parallelogram*, is found by multiplying its base into its perpendicular. Thus let the accompanying figure be 10 chains in length, and 2 in width; the area, or content, will be 20 square chains, or 2 acres.



It is evident that the area of a square being given, its square root will be the length of one side; and

The area of a parallelogram in any denomination, as chains or perches, and the length of one side in the same denomination being given, divide the area by the given side, the quotient will be the length of the adjacent side.

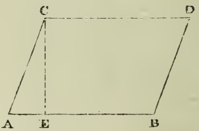
PROBLEM II.

To find the area of a rhombus, or oblique-angled parallelogram.

Multiply the base by the perpendicular height. The reason is plain from Euc. El. B. 1, prop. xxxv.

Example.

Let A B C D be a piece of ground in the form of a rhombus, whose base A B = 22 chains, and perpendicular C E = 20 chains. Required the area or content.



22
20
——
44.0 Acres.

If the area in any denomination, and one side in the same denomination were given, to find the perpendicular; it would be done by dividing the former, by the latter, thus:

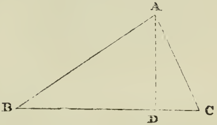
22)440 square chains
20 = perpendicular.

PROBLEM III.

To find the area of a triangle.

Multiply the base by half the perpendicular height, or the perpendicular height by half the base; or take half the product of the base into the perpendicular.

The reason is plain from Euc. El. B. 1, prop. 41. Let A B C be a triangular piece of ground, whose base, or longest side is 12.38 chains, and perpendicular height A D, is 6.78.



Base 12 38
½ perp. 3.39
11142
3714
3714
Acres 4.19682
4
Roods .78728
40
Perches 31.49120

A. R. P.
Area 4 . 0 . 31.

The other methods are obvious, but should be worked by the learner, for his own satisfaction.

To construct the figure, it is evidently necessary to have the distance given from one end of the base, to the perpendicular.

PROBLEM IV.

The content of a triangular piece of ground, and one side as base, given, to find the perpendicular.

Divide the area in the given denomination, as perches, by half the base, in the same denomination, the quotient will be the length of the perpendicular.

This perpendicular being set out on any part of the base, and lines run from its extremity to the ends of the base, will lay out the triangle, (Euc. El. B. 1, prop. 35.) so that the perpendicular may be set on that part of the base, which is most desirable.

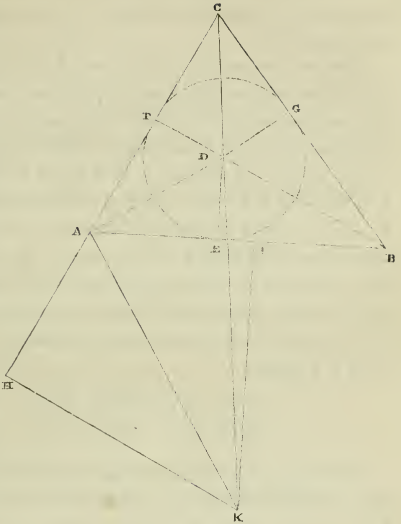
PROBLEM V.

To find the area of a triangle, as A B C, (next fig.) having the three sides given.

From half the sum of the lengths of the sides, take each side separately; and multiply half the sum of the sides, by the three remainders continuously; the square root of the product will be the area of the triangle.

Bisect any two of the angles, as A and B, with the lines A D, B D; draw the perpendiculars D E, D F, D G.

The triangles A F D and A E D are mutually equiangular; for the angle F A D = E A D by construction, and A F D = A E D, being both right angles; consequently the remaining angles A D F and A D E are equal; wherefore, A D : D F : : A D : D E; and since A D bears



the same proportion to both D F and D E, D F = D E; and the triangle A F D = A E D. In the same way D E = D G; therefore D will be the centre of a circle which will pass thro' E, F, G.

In the same way if A and C were bisected, the same point D would be had; therefore, a line from D to C will bisect C, and thus the triangles D F C and D F G will also be equal.

Let C A be produced to H, making A H = E B or G B; so will H C be equal to half the sum of the sides, that is to $\frac{1}{2}$ A B + $\frac{1}{2}$ A C + $\frac{1}{2}$ B C; for F C, F A, E B, are severally equal to C G, A E, B G; and all these together are equal to the sum of the sides of the triangle; therefore, F C + F A + E B, or C H, is equal to half the sum of the sides.

F C = C H — A B, for A F = A E, and H A = E B;
therefore, H F = A B;

and A F = C H — B C; for C F = C G, and A H = G B;
therefore, B C = H A + F C, and A H = C H — A C.

Let C D be produced to meet a perpendicular erected upon H, in K; and from K draw the perpendicular K I; also draw A K.

Because the angles A H K and A I K are both right angles, the angles H A I and K together, are equal to two right angles, for all the angles of the quadrilateral, or the two triangles into which it is divided, contain four right angles; in the same way F D E + F A E (= 2 right angles) = F A E + H A I; let F A E be taken from both, then F D E = H A I; and of course F A E = K; the quadrilateral figures A F D E and K H A I, are therefore similar, and consequently have the sides about the equal angles proportional; and it is evident the triangles C F D and C H K are also similar, because F D is parallel to H K; hence

$$FD : HA :: FA : HK$$
$$FD : FC :: HK : HC$$

Wherefore by multiplying the extremes, and means in both, it will be, $FD^2 \times HK \times HC = HA \times FA \times FC \times HK$; take H K from both, and multiply each side by C H; then $FD^2 \times CH^2 = HA \times FA \times FC \times CH$.

Hence it is evident that $\frac{1}{2} AB \times DE + \frac{1}{2} BC \times DG + \frac{1}{2} AC \times FD$ = the area of the triangle; or that half the sum of the sides, viz. C H, × F D = the area. Wherefore, $CH^2 \times FD^2 = FC \times FA \times HA \times CH$; that is, the half sum multiplied continually into the differences between the half sum and each side, will be the square of the area of the triangle, and its root the area. Q. E. D.

Example.

Let A B = 1064 links.
A C = 1228
B C = 900

Sum = 3192

Half sum = 1596 — 1064 = 532 first remainder.
,, — 1228 = 368 second ,,
,, — 900 = 696 third ,,
1596 half sum.
532 1st remainder.

849072
368 2nd remainder.

312458496
696 3rd remainder.
$\sqrt[2]{217471113216} = 466337$ area in square links.

10000) 466337 (4.66337 Acres.
4

Roods 2.65348
40

Perches 26.13920

	A.	R.	P.
Ans.	4	2	26

To perform this by Logarithms.

Inasmuch as the addition of logarithms answers to the multiplication of their corresponding numbers; and as the number answering to the half of a logarithm will be the square root of the number of that logarithm, it follows that, Half the sum of the logarithms of half the sum of the sides and the three remainders, will give the logarithm of the area. Thus

	Chains.	
Half sum	15.96..	1.2030329
1st. Remainder	5.32..	0.7259116
2nd. Do.	3.68..	0.5658478
3rd. Do.	6.96..	0.8426092

2) 3.3374015

1.66870075..46.634 Sq. Chains.

Acres 4.6634
4

Roods 2.6536
40

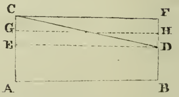
Perches 26.1440

A. R. P.
Ans. 4 2 26 as before.

Problem VI.

To find the area of a trapezium, as A B C D.

Multiply the base, as A B, by half the sum of the perpendiculars, as A C, D B, or the two perpendiculars by half the base; or take half the product of the base into the sum of the two perpendiculars, the product will be the area.



Produce B D to F, so that B F equals A C; from D draw D E parallel to A B, and from C draw C F also parallel to A B. It is evident that the parallelogram C E D F is equal to double the triangle C E D, and of course that the triangle is half the parallelogram; it is also evident that E C is the difference of the perpendiculars A C, B D. Bisect E C in G and draw G H parallel to A B. Now it is evident that A G is equal to half the sum of the perpendiculars, for it is the greater quantity minus half the difference.

Also it is manifest that the parallelogram G E D H is half of C E D F; but the triangle C E D is also half of C E D F, therefore, the triangle C E D is equal to the parallelogram G E D H; therefore the area of A B G H = A B C D. Q. E. D.

Example.

In the same figure let the length of A B = 6.60 chains, A C = 1.50 and B D 80 links.

1.50 }
.80 } Perpendiculars.

2.30 sum.

1.15 half sum.
6.60 base.

6900
690

7.5900 Square chains.
4

Roods 3.036 A. R. P.
40 Ans. 0 3 1

Perch 1.440

The other methods are obvious, and when the sum of the perpendiculars is an odd number, so that the half cannot be conveniently taken, take half the base, and the whole sum of the perpendiculars ; if both, the sum of the perpendiculars, and the base, are odd numbers, multiply one by the other, and take half their product, for the area. It will be perceived that this problem applies particularly to the calculating of offsets from the field-book.

Content of Land.

A parish or other portion of land having been surveyed in the manner already described, it is evident that the area may be determined " independent of the result obtained by summing up the contents of each inclosure," and in " fact to arrive at the total area by direct admeasurement of the space included within its external boundary." Thus in fig. I, pl. 1, having divided the parish into the large triangles, A O C, C O B, B O D, D O A, A F E, and the minor ones T T T T, their areas may be ascertained from the measured lengths of their sides, by Prob. V. of this section ; adding thereto the several offsets coloured pink, calculated by Prob. VI.; and subtracting from the sum, the two triangles T′ T′, coloured blue and yellow, together with the several offsets, also coloured blue and yellow, which fall within the measured lines, calculated likewise from their admeasurements. It is manifest that, to check the calculation of the large triangles, the figure may be taken in two triangles, as A C D and B C D. The following form of content register, adapted from that in use on the Irish survey, may be adopted with advantage.

CONTENT REGISTER.

	Triangle or Trapezium.	1st side.	2nd side.	3rd side.	Content in Chains.	Content in Acres.
Additive	A C O	170.50	151.50	75.00	5679.90	
	A E F	110.00	32.00	112.00	1754.00	
	A E	—	6.00	31.00	93.00	
		6.00	5.00	13.50	74.25	
		5 00	6.00	28.50	156.75	
		6.00	3.00	4.00	18.00	
		3.00	2.00	13.00	32.50	
		—	1.00	16.00	80.00	
		3.00	—	8.00	12.00	
	F C	4.00	.50	12.00	54.00	
		.50	.50	11.00	5.50	
		.50	5.00	1.00	2.75	
		5.00	4.50	8.00	38.00	
					8000 65	
Negative	F C	4.50	4.00	15.00	63.75	
		4.00	1.00	6.00	15.00	
		1.00	4.00	3.00	7.50	
		4.00	2.00	3.50	10.50	
		2.00	1.50	11.50	20.12	
		1.50	4.00	10.00	27.50	
		4.00	—	9.50	19.00	
			Total Negatives		163.37	
			Total Additives		8000.65	
					783.7.28	
					4	
					2.912	
					40	
					36.480	783 2 36

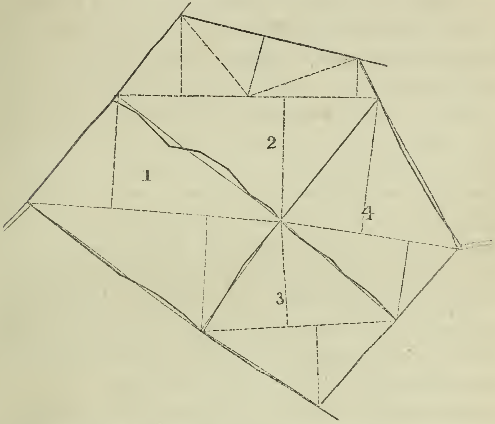
It is not considered necessary to go through the computation of the whole figure, as the principle, it is hoped, will be readily understood from the foregoing example. It will be perceived that the offsets are very much greater than could be admitted in practice; this arises from the necessity for distinctly exhibiting the lines on a scale within publishing compass. It may be observed, that to obtain the whole area, contained within the external boundary, from actual measurement, it is necessary to obtain the whole of that boundary from the external lines; no offsets should therefore be taken from the internal ones, as shown at *a a* on O B.

It is evident that the internal areas might be obtained in a similar manner, from actual measurement; or, the work having been duly plotted on a large scale, the plan might be advantageously divided into triangles, the offsets reduced to regular figures, (by prob. xviii. Section i.) and the necessary lengths of sides taken from the paper; indeed this latter is, in part, the mode adopted on the Irish Survey. The reader who desires a more elaborate example of this system of computation, is referred to Lieut. Frome's work, already mentioned.

The modes for computing, or casting, areas, generally practised by civil surveyors are the following, each having its advantages, and being applicable according to circumstances.

1st. The plan having been plotted on a large scale, as 2, 3, 4, or 5 chains to an inch, the several inclosures are divided into triangles, two of which triangles frequently of course form a trapezium, as 1, 3, and 4, in the accompanying figure, and may be calculated accordingly, by taking the two perpendiculars in one; the contents of these figures are then computed by their bases, and perpendiculars, measured on the paper, by the same scale with which the plan has been plotted.

It may be necessary to say, that the content, or working plot, ought not to contain any ornamental or extraneous matter whatever. The boundaries of the several enclosures should be accurately plotted, finely ruled in, and not



drawn by hand, and then each line carefully equated, or balanced, by a give-and-take line for computation.

Lines may be balanced by the parallel ruler as already mentioned; or by the eye, with a horse hair stretched on a small bow of whalebone; or by a transparent horn, prepared for the purpose, either the hair or the horn being applied to a crooked line. The position of a straight one is to be determined by the eye, which shall give as much in one part, as it takes in another. Sometimes the first of these modes may be advantageously combined with one of the others; if numerous offsets have been taken, the boundary may be equated in short lengths, by the hair or the horn and afterwards those lengths reduced to one line, by means of a parallel ruler.

As has been elsewhere observed, the triangles should be of as good a shape as possible, that is, not having any of the angles either very acute, or very obtuse; otherwise material errors may arise in measuring to them.

A content plot should be kept as nearly as possible, in the same hygrometrical state, until it has been computed; if it be plotted in a cold room, in damp weather, and computed in a warm dry room, the paper will contract considerably, or if, on the contrary, it be plotted in a dry atmosphere, and computed in a humid one, the lines will of course be elongated. If a survey is extensive, so that the plan may be laid down in summer, and computed in winter, or *vice versa*, a scale should be laid down on the paper, and the distances taken from it in all cases, as such a scale would evidently suffer the same vicissitudes from the weather, as the plan itself. The " Engine divided scales," engraved on pasteboard, lately introduced by Mr. Holtzapffel, will be a near approximation to this, and will probably be found sufficiently so for all ordinary purposes.

Each subdivision should be distinguished in the plan, in the content register, and in the reference or schedule, by the same number; these numbers should follow each other consecutively on the plan, to facilitate reference.

2nd. The whole plan may be reduced into small squares, or parallelograms, by drawing parallel lines throughout it, at regular intervals; and crossing those parallel lines by others, at right angles to them, and at the same distance asunder. Thus if a plan has been plotted at 4 chains to an inch, and parallel lines to be drawn through it, at every $\frac{1}{4}$ of an inch, crossed by similar lines at right angles, the plan will evidently be divided into square chains; the crooked lines being then made straight, and the squares carefully counted, the content of the whole, or of the several parts, may be had in square chains.

The best mode of computing fractions of squares, is to have a piece of transparent horn, with one square scratched on it of the same size of those on the plan ; and that square subdivided into 100 other squares, by drawing 10 parallel lines through it, in one direction, and crossing them by 10 others; each small square will of course, be $\frac{1}{100}$ part of a large one, and may be set down decimally with the large squares, and treated accordingly. Observing if there are less than 10 small squares to set them down as hundredths, and not as tenths. Or if one square chain on the horn be divided into 16 parts, by drawing three parallel lines through, in both directions, each small square will represent one perch, and must be set down in a distinct column.

3rd. By the scale of acres. This is much used by some surveyors, particularly in Ireland. It is a scale of equal parts; each large division being 8-10ths of an inch in length; having one of those large divisions subdivided into tenths, or diagonally into hundredths.* A plan then being plotted at 5 chains, or 20 perches to an inch, and having parallel lines drawn throughout it, horizontally, half an inch asunder, each large division on the scale will represent one acre, between each pair of parallels; because it will be 16 perches in length and 10 in width, and there are 160 perches in an acre; the small divisions will be decimals. The end of a rectangular transparent horn, half an inch in width, with a line drawn on it longitudinally, at half its width, being brought to a line that cuts any two parallels obliquely, bisects it, and consequently by this means all right lined figures may be reduced to parallelograms. If a fence does not cut any two of the parallels, the fractional part may be estimated by a square horn, as before; or computed as a triangle, by the base and perpendicular.

In all these cases, much care is requisite, to avoid either taking a sub-division twice, or omitting one altogether.

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* Vide, p. 34.

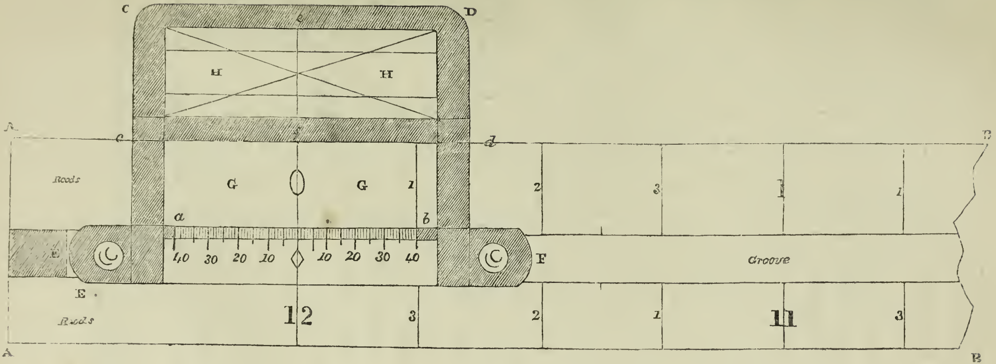
The parallel lines may be drawn, either in a faint colour or with a hard pencil.

MARQUOI'S PARALLEL SCALE.

The instrument by which parallel lines may be drawn with the greatest accuracy and facility, is Marquoi's parallel scale. These parallel scales consist of a right angled triangle, whose hypothenuse, or longest side, is three times the length of the shortest; and of two rectangular rulers, each usually twelve inches in length. They answer every purpose of a parallel ruler, together with the advantage of erecting perpendiculars, to any part of a line. Parallel lines may be drawn accurately at any required distance, without the aid of compasses, by means of the artificial scale on the edge of the ruler, each division on which is three times the length of its corresponding natural scale underneath. These natural and artificial scales bear the same proportion to each other as the shortest perpendicular does to the hypothenuse of the triangle. The divisions on the rulers are numbered both ways, from the centre; and on the triangle there is an index or zero point, which must be placed in coincidence with that on one of the rulers, and then, by sliding the triangle along the face of the ruler, lines may be drawn parallel, on either side of a given line, at any distance, pointed out by the index on the triangle. When it is required to draw very close parallels, the artificial scales will greatly assist the sight.

4th. But the best mode of casting contents is by a *Computing Scale* lately introduced.

COMPUTING SCALE.



This scale consists of a box ruler or scale, as A B, fig. 70, of any required length, and a slide, C D E F. The slide works in a groove in the scale; the space G G is open in the slide, H H contains transparent horn, the remainder of the slide is of metal. The scale represented in the sketch is adapted for computing plans of three chains to an inch. Suppose a plan plotted to that scale, and parallels drawn horizontally throughout it 1-3rd of an inch asunder. As there are 4 roods, and also 10 square chains, in an acre, it is evident that 2½ square chains equal 1 rood; the divisions on the scale are therefore 2½ chains in length; consequently each one represents a rood; and on the slide, at each side of the zero or index, is a division equal to one of those on the scale, divided into 40 parts, each part is therefore one perch. At *c d* on the slide there is a step formed, of the thickness of the scale, by means of which the portion *c d* C D of the slide works close to the paper. In the sketch, the instrument is represented at zero.*

To compute the content of a plan, let the scale, as here represented, be applied with the parallel lines, at H H, on the horn, coinciding with the two first parallels on the paper, and the line, *e f*, corresponding with the zero on the scale, constituting a give-and-take line with the first boundary. Now let the scale be held fast, and the slide be moved until the line *e f* constitutes a give-and-take line with the next fence; then, without disturbing the slide, remove the instrument, bodily, down to the next pair of parallels, making *e f* cut the left hand boundary, and the parallels at H H coincide with those on the plan, as before; again move the slide, until *e f* cuts the first fence on the right; without disturbing the slide, proceed to the next pair of parallels; and so on through the whole enclosure, or until the end of the scale has been arrived at. The scale, of which the sketch represents one end, is adapted to measure 12 acres; that is, 6 each way; so that having measured 6 acres from left to right, other 6 may be computed from right to left, without disturbing the slide; or the slide may be taken out of the groove, and turned to the other side of the scale, and so the computation be still carried on from left to right. Then, if an inclosure contains more than 12 acres, the scale must be recommenced; but if less than twelve, the slide having been duly, as above directed, passed over all its parallels, the area is read off at once. Suppose, for instance, the slide had been passed through the parallels of an enclosure until its zero had reached the figure 6 on the scale, and then back again, until the zero stood half way between 1 and 2, to the left of the large 11; the division marked 20, on the small scale, would then coincide with that marked 1 on the large one, and the quantity would be 11 a. 1 r. 20 p.; the large figures denoting acres, the smaller ones roods, and the smallest perches.

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* The scale here described has been modified, in some respects, by Mr. Elliott, Optician, Holborn. I regret that I have been unable to ascertain who was the original inventor.

Portions falling between two parallels may be estimated by a transparent horn, as in the other cases.

A substance called glass paper has been lately introduced, on which parallels may be drawn, to avoid drawing them on plans. The substance is quite transparent, and not expensive.* Having then fixed a sheet of such paper, duly divided, in a proper position on a plan, the computation may be proceeded with as if the plan itself had been ruled. It is evident that tracing paper would answer the same purpose, but would not be so durable.

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* Sold by Newman, Soho Square, and others for 1*s* per sheet.

Fair Plans of Estates, &c.

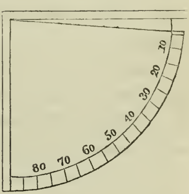
The degree of finish to be given to the fair plan will depend upon circumstances.

To plans of Estates a high degree of finish may be given; everything recommended to be shown on parish plans, may be shown also on these, together with hills, &c. All the hills in one plan ought to be delineated by one hand, to ensure some degree of uniformity in the shading. The depth of shading should be in proportion to the angle of inclination, and not to the height of each hill. Hills may be roughly sketched in the field-book; but a more satisfactory way is to take a tracing of the plan and go over the ground with it, for the purpose of sketching hills, &c. Many minor matters, as remarkable trees, hedgerows, small ponds, &c. may be sketched at the same time. "In addition to the sketch of the ground, a representation of the geological features of the country is given by several of the gentlemen who fill in the outline plan of the survey of Ireland, without at all interfering with, or confusing the sketch. On the back of the paper are traced the divisions of the geological features, and these different portions are coloured according to the conventional system of distinguishing the various formations on geological maps. On holding the sketch against the light, these divisions appear clearly visible, though in any other position of the paper they are not in the least perceptible. Geological sections are also shewn on the margin of the sketch, having reference to lines drawn across it."

Clinometer.

"The inclination of such slopes as are of peculiar moment should be measured with a Clinometer, fig. 71, and the angles written either on the slopes themselves, or as references. This little instrument may be made by cutting a small quadrant out of pasteboard, and roughly graduating the arc. A small shot, suspended by a piece of silk, forms the plummet. The instrument sold under that name is made with a spirit level; but the substitute, as shown in the sketch answers the purpose equally well, and moreover, from its being made merely of pasteboard, fits into the pocket of the sketching portfolio."

" The leading features of ground are, the summit ridges of hills (sometimes termed the water-shed lines), and the lowest parts of the valleys, down which the rain finds its way to the nearest rivers or pools, called water-course lines. These two directing lines if traced with care, will alone give some idea of the surface of the country, and assist materially in sketching the hills."*



The outline of the curves, &c. having been sketched in the field are transferred to the plan, to be shaded. This is generally finished with a brush, either in India-ink, or Sepia; but a great want of one general system of topographical plan-drawing is felt; particularly as regards the method of expressing the features of the ground in a manner at once easy of execution, and generally intelligible.

In topographical plan-drawing as now practised, the light is considered as falling nearly vertical, but sufficiently oblique to allow of a decided light and shade to the slopes of the hills, trees, &c. The hills are shaded, not as they would really appear in nature, but on the conventional system of making the slopes darker in proportion to their steepness; the summits of the highest range being left white.

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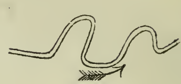
* Lieut. Frome.



Hedges should be sketched, either with a pen or a camel's hair pencil, on the side of the boundary to which they belong. Brooks, water-courses, drains, &c. should be drawn with a double line, with small arrows to denote the direction of the current.

Some surveyors give an elevation of the Manor-house in the body of the plan, but this is very objectionable; the ground-plan only should be shown, and that plotted on the same scale as the other portions of the plan. A view of the proprietor's seat may, very appropriately, be shown in one corner of the paper.

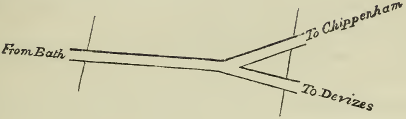
It is usual, on the finished plan, to lay down the magnetic meridian, and to embellish it. A line purporting to be the *true north* line, is also frequently shown in conjunction with the magnetic, but this ought not to be done unless it has been properly ascertained; it is much more honest to omit it altogether, than to put it in inaccurately.

The scale by which a plan has been laid down should also be neatly drawn.

The title should be handsomely written in one corner, when the whole survey is contained in one sheet; but if the survey extends over several sheets, these must be formed into a book or Atlas, with an appropriate title-page, &c.

The names of all circumjacent lands, or of their proprietors, should be written in their proper positions; and all abutting fences shown, especially boundaries.

The direction of roads represented on the plan should be shown, thus—



The embellishing and colouring of maps is not so much the business of the practical surveyor as of the professed draughtsman, whom it is not pretended here to instruct.

The fair plan may either be plotted anew, or copied from the content plot. The most expeditious mode of copying plans is, being provided with a piece of plate glass of adequate size, properly fitted into a frame on a stand, at a convenient slope, the plan to be copied being placed on the glass, with the plain paper over it, both are pinned to the frame, when a strong light being admitted to the back of the glass, the plan may be copied with great facility and correctness. This may be used even at night by placing a good lamp at the back of the frame.

If a glass and frame are not at hand, or if the plan is too large, it may be pricked off with a fine needle. It is not unusual to prick off the fences, and other details, but it is much better to prick off the station lines, and to plot the work anew. If any portion should have been omitted in pricking off, it may be traced off on tracing paper, together with a few fixed points, and so transferred to the fair plan.

The whole plan might be transferred, by copying it on tracing paper, and then having laid down a piece of thin paper evenly blackened at the back, either with blacklead, or burned cork, on the fair paper, and the tracing over all, the lines may be gone over with a fine, but not a sharp point,* and so transferred. It is possible to transfer a plan directly, by placing blackened paper in a similar manner, between it and the other paper; but manifestly to the injury of the original plan.

When a plan is to be copied upon a different scale to that by which it has been laid down, other means must be resorted to, which are also applicable to copying upon the same scale. A plan being divided into a number of small squares, say one chain each, and the paper to which it is to be transferred, divided into a similar number of squares, in the proportion of the required scale, the horizontal and vertical lines may then be distinguished by corresponding figures in both, and the details of the plan transferred with considerable accuracy; due care being of course taken. If it is not admissible to draw squares on the original plan, they may be drawn on tracing paper, and properly fastened in their position.

The Pentagraph† is frequently used for copying, or reducing plans, but should not be used for enlarging, as already mentioned.

To Plot a Chain Survey.

Enough has been said, to enable the young surveyor to lay down his work, when made with the chain only, as already described; he must however be careful to have his straight-edge truly straight; and to lay down all his distances by the compasses, and not trust to measuring by the scale. When the distances are too great to be taken at one length in common compasses, beam compasses should be had recourse to; as errors are inevitable if distances be stepped in several lengths.

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* A hard pencil, a porcupine's quill, or the eye end of a needle will answer the purpose.

† See Simms, p. 112.

In laying down a triangle, it is convenient to use two pair of compasses, so that having marked off the base, the length of both the other sides may be taken, each in different compasses; then one being held in each hand, with a point at its appropriate end of the base, the point of intersection may be found without stretching or losing time.

The station lines being laid down, the offsets are to be plotted. If numerous offsets have been taken, several of their distances may be set off without removing the scale. To effect this the commencement of the scale is to be made to coincide with the commencement of the line, and the edge of the scale with the line itself; when, as is evident, the chained distances may be marked, as they were taken on the ground. The scale is then to be turned at right angles to the line, as already directed, and a few offsets marked; if more than three or four are marked at once, without drawing in the line which connects them, some of them are very liable to be overlooked. It is scarcely necessary to say, that the line, connecting the points so laid down, will represent the line to which the offsets have been taken on the ground.

Two or three modes of facilitating the plotting of offsets have been invented, and applied on the Ordnance Survey, but civil surveyors generally work to such large scales, that none of these modes are applicable to their practice.

Incorrect Chain.

If a surveyor should discover that he has made a survey, or any part of one, with an incorrect chain, he ought to go over the work again; because it is not likely that his chain has attained the maximum incorrectness all at once. But if he is satisfied that the chain has not materially altered during the progress of the work; or if the survey is not of sufficient consequence to induce him to go over it again, he may make the correction in the house, thus:

To correct the length of a line measured with an inaccurate chain, it will be,

As the length of a true chain,
Is to the measured length of a given line,
So is the length of the incorrect chain,
To the true length of the given line.

OR,

To find the true content of a survey, taken by an incorrect chain, the work being plotted; it will be,

As the square of a true chain,
Is to the calculated content of the plan,
So is the square of the incorrect chain,
To the true content.

TO REDUCE ONE MEASURE TO ANOTHER.

The reduction of Customary, Plantation, or other local measure, to Statute measure, and the contrary, is accomplished by the application of the same rules as the correction of the chain Thus the length of a Plantation perch is 7 feet, and of a Statute perch $5\frac{1}{2}$ feet; but, for the convenience of working in whole numbers, let both these numbers be doubled, and we shall have 14 and 11; then to reduce statute measure to plantation measure, say a lineal distance of 10 miles; it will be,

Example I.

feet miles feet
14 : 10 : : 11
1760
17600
3
52800
11
)———(41485.714
580800
56
20
14
68
56
120
112
80
70
100
98
20
14
60
56
.4

5280)41485.714(7.857
36960 8
45257 6.856
42240 40
30171 34 240
26400
37714
36960
. . 754

m. furl. poles.

Ans. 10 miles Statute measure = 7 6 34 Plantation measure.

Example II.

To reduce 10 miles Plantation measure to its equivalent value in Statute measure, it will be,

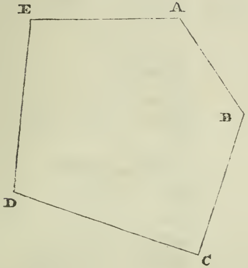
```
       feet.      miles.     feet.
        11    :    10  : :   14
                  5280
                 ------
                 52800
                    14
                -------
                211200
                52800
                -------
            11) 739200
                -------
      528,0)    6720,0(12.727
                528          8
                ----     -----
                1440     5.816
                1056        40
                ----    ------
                 3840   32.640
                 3696
                 ----
                  1440
                  1056
                  ----
                   3840
                   3696
                   ----
                    144
```

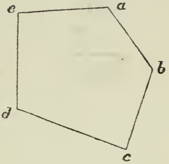
m. furl. poles.
Ans. 10 plantation miles = 12 5 32 statute measure.

It is evident that if any parcel of ground be measured by chains of different lengths, that the figures, constructed from these different measures, will be unequal, but similar;

and therefore such figures will be in a duplicate proportion to their homologous sides, by prop. xx. B. vi. Euc. El.

Let *a b c d e* be a plan of a piece of ground measured by a plantation chain; and A B C D E a plan of the same





piece, measured by a statute chain; if the line *a b* in the one be 11 plantation perches, the corresponding, or homologous, side, A B, in the other, will be 14 perches statute measure, for the reason already mentioned. It is evident then, that

1. To reduce Plantation acres to Statute acres, it will be

As the square of 11, or 121,
Is to the area in plantation acres,
So is the square of 14, or 196,
To the area in statute acres.

2. And to reduce Statute acres to Plantation acres, it will be

As to the square of 14, or 196,
Is to the area in statute acres,
So is the square of 11, or 121,
To the area in plantation acres.

Example I.

Reduce 246 a. 3 r. 14 p. Plantation measure, to Statute measure.

4
987
40
39494

121 : 39494 : : 196
196
236964
355446
39494
) 7740824 (63973
726
480
363
1178
1089
892
847
454
363

A. R. P.
160) 63973 (399 3 13 Statute measure.
480
1597
1440
1573
1440
4,0) 13,3
3.13

Example II.

Reduce 416 a. 2 r. 14 p. Statute measure, to Plantation measure.

```
         4
     ------
      1666
        40
     ------
196 : 66654 : : 121
        121
     ------
      66654
     799848
     -------                              A.  R.  P.
)8065134(41148          160)41148(257  0  28
  784                        320
 ----                       ----
  225                        914
  196                        800
  ----                      ----
   291                       1148
   196                       1120
  ----                      ----
   953                        28 perches
   784             A.  R.  P.
  ----        Ans. 257  0  28 plantation measure.
   1694
   1568
   ----
    126
```

It is not considered necessary to give examples of the reduction of Scotch, Customary, or other measures, because the same principles apply to all; merely taking care to make use of the appropriate number of feet, or other low denomination, contained in the perches of the respective measures to be compared.

SCALE OMITTED.

Should a surveyor be required to ascertain the scale by which a map has been laid down, of which the area is given, but the scale omitted.

Cast up the map by any assumed scale, and it will be,

As the area found,
Is to the square of the assumed scale,
So is the given area,
To the square of the scale by which it was laid down.

Thus let the given area of a map be 126a. 3r. 16p.; and let it be cast up by a scale of 20 perches to an inch, and the area thereby produced be 31a. 2r. 34p.; then say,

As the area found, 31a. 2r. 34p.
Is to the square of 20 = 400
So is the given area, 126a. 3r. 16p.
To the square of the scale by which it was laid down;
which will be found to be 40 perches to an inch.

DISPUTED BOUNDARIES.

So many disputes have been settled during the progress of the Ordnance, and Tithe Commutation surveys, that it is presumed few will remain to be settled, in this kingdom, by future surveyors; nevertheless, as some disputes must still arise, from one cause or another, the surveyor should be acquainted with a mode of settling them.

The first thing to be done is, to ascertain, from competent persons, on the ground, the boundary claimed by each party; then to make a survey of the disputed portion, together with so much of the adjoining boundaries as will enable him to compare the new survey with such old maps as may be in possession of the respective parties. If an authentic map can be found, sufficient data will have been obtained, for the settlement of the dispute. For let the new survey be laid down by the same scale as the old map, and let an accurate tracing of one be applied to the other, when the difference will immediately be apparent; which, if the old map is to be adhered to, may of course be readily marked on the ground.

But the settlement of disputes respecting private property frequently depends in a great degree on *possession;* which is a question of law, and not of measurement.

If no authentic maps of the disputed property exist, and the parties cannot be induced to agree to a line, the surveyor must reduce the disputed portion within the narrowest possible limits, and then divide the difference; having regard to the difference of value, if any; if there is no difference of value, then let the difference of quantity be equally divided between the parties.

INCLOSURES.

When a mountain, a moor, a bog, or a common, hitherto held in common, is to be divided, the amounts of the claims are decided, either by the extent and value of the respective adjoining lands, or by the extent of their frontage to the common land. In either case, if good maps of the several properties on which the claims are founded, can be obtained, they should be reduced to one common scale, and formed into one common map. The whole of the land to be divided being then surveyed, and the survey laid down on this common map, the surveyor will be enabled to make his allotments on paper. If maps of all the properties are not to be had, then the survey must include so much of the boundaries of the properties of which maps are wanting, as will enable the surveyor to make an equitable allotment. So much depends upon the shape of the mountain, moor, &c. and of the adjoining lands; and on the value of both, that it would not be possible to give any definite general instructions: but to give satisfaction to the several claimants will require the exercise of the utmost discretion of a very discreet and competent surveyor; and none other should attempt it.

In cases where the proportional amount of the several claims on a common is defined, by its having been settled what number of head of cattle the representatives of the respective properties are entitled to graze, the first thing to be done is, to make a valuation of the land to be divided.

The valuer or valuers, being provided with a labourer and spade, &c., should walk over the ground, and ascertain the bounds of the different qualities of the land, and mark such boundary lines on the ground. The whole should then be surveyed; the portions of different values, sometimes called *flats*, being measured as inclosures; the lines already mentioned, sometimes called *quality lines*, of course representing fences; but shown either as dotted lines, or in coloured ink.

To value land satisfactorily requires much experience. The principal points to be considered are, the quality and depth of soil, the nature of the sub-soil, that is, whether of limestone or other rock, of gravel or sand, of clay, peat, &c.; the situation, with regard to markets, and facility of transport, whether by road or by water; whether well drained and watered, or subject to periodical inundation, &c. Where a road runs through a common, the contiguous land may become building ground, which will of course enhance the value very considerably. If there is no material difference of value, an average may be struck for the whole.

To avoid confusion, much care is required in keeping explicit accounts. Having prepared an alphabetical list of claimants, with the number of shares to which each is entitled, it is evident that the total number of claims may be had. A book, which may be called the *Flat book*, may then be prepared; each page being divided into five columns as follows, viz.

Claimants.	Flats.	Quantities.	Value per acre.	Annual value.

Suppose then the total quantity and value of every flat to have been ascertained, and the amount of value expressed in shillings and decimals, in the 5th column. All new roads, gravel pits, drains, &c. should be first set out, and the quantity of each flat so occupied, deducted from its total amount, and the appropriate entry made in each column. The manorial allotment should next be set out; this generally amounts to from 1-16th to 1-20th of the whole common. An allotment in lieu of tithes should next be made; which is far preferable to leaving the whole subject to tithe, by which future improvements are materially impeded. The remainder is to be divided between the land claimants, and the common-right houses. The whole being thus disposed of, the several parts should of course, equal the whole, of each flat; and the quantities in the several flats equal the total quantity in the common.

TO FIND THE VALUE OF COMMON RIGHTS.

Divide the total value to be allotted, by the total number of rights, which will give the value of one right; and that multiplied by each claimant's number of rights, will be the amount to be allotted to each, respectively.

TO FIND THE PROPORTIONAL VALUE FOR LANDS.

Divide the total value to be allotted, by the total value of the estates, and the quotient will be a common multiplier; then the value of each estate multiplied by the common multiplier, will be the proportional value to be allotted to each, respectively.

Example.

Divide a common of 4500 shillings value, between the estates X, Y, Z; X estate is valued at 1200, Y at 1800, and Z, at 2000 shillings.

1200 X	1200×.9 = 10800 =	X's share.
1800 Y	1800×.9 = 16200 =	Y's do.
2000 Z	2000×.9 = 18000 =	Z's do.
5000) 4500.0 (0.9	45000	Total.
45000		

EXAMPLE OF AN ENCLOSURE.

Let it be required to allot a common, containing fifty acres, according to the following claims, viz. For a new road 60 feet in width, including fences, and 30 chains in length. W. Earle, Esq. as lord of the manor, one twentieth part of the whole. The Rev. Thomas Goodchild, as rector, one sixth, in lieu of rectorial and vicarial tithes. John G. Thompson, 5 common rights, John Oldham, 4 do. and W. Augustus Treacy, 1 do.; in all 10 rights.

The common has been divided into 4 flats, as follows, viz.

Flats.	Contents. Acres.	Value per Acre.	Total Annual value.
N°. 1	15.00	30	450.00
" 2	10.25	20	205.00
" 3	20.50	40	820.00
" 4	4.25	15	63.75
	50.00		1538.75

The proposed road will occupy the following portions of the several flats, viz.

	length in chains.		width in links.	Contents.		Value p. acre.		Annual Value in shillings.
N°. 1	7.50	×	90	.675	×	30	=	20.250
" 2	9.00	×	90	.810	×	20	=	16.200
" 3	5.30	×	90	.477	×	40	=	19.080
" 4	8.20	×	90	.738	×	15	=	11.070
Total chains	30.00		Total acres	2.700				66 600

A portion of No. 2 flat, value 12 shillings, is to be reserved as a gravel pit. The value of the quantity to be allotted will be therefore,

		s.
	Total value	1538 75
Deduct for road	66.600	
" for gravel pit	12.000	78.60
	Total to be allotted	1460.15

20) 1460.15

73.007 = Manorial right

6) 1460.15

243.36 = Rector's right.

	Total value	1538.75
Deduct for Road	66 600	
" Gravel pit	12.000	
" Manor right	73.007	
" Rector's right	243.360	
	———	394.967

Remains to be divided 1143.783 between the other claimants.

10) 1143.78

114.38 = One common right.

114.38 × 5 =	571.90	John G. Thomson
114.38 × 4 =	457.52	John Oldham
114 38 × 1 =	114.38	W. A. Treacy
	73.07	W. Earle, esquire
	243.36	Rev. Thomas Goodchild
	78.60	Road and Gravel pit
Total value =	1538.83	

Book of Flats.

Claimants.	Flats.	Quantities.	Value per Acre.	Annual Value.
Proposed Road	- -	.675	- -	20.25
Lord of the Manor	- -	1.750	- -	52.50
John G. Thomson	- -	12.575	- -	377.25
	1	15.000	30	450.00
Road	- -	.810	- -	16.200
Gravel pit	- -	.600	- -	12.000
Lord of the Manor	- -	1.003	- -	20.050
The Rector	- -	7.837	- -	156.750
	2	10.250	20	205.000
Road	- -	.477	- -	19.080
Rector	- -	2.165	- -	86.500
John G. Thomson	- -	4.866	- -	194.650
John Oldham	- -	11.438	- -	457.520
W. A. Treacy	- -	1.554	- -	62.250
	3	20.500	40	820.000
Road	- -	738	- -	11.070
W. A. Treacy	- -	3.512	- -	52.680
	4	4.250	15	63.750

ALLOTMENT BOOK.

Finally the allotment book should be made out, arranging the claimants alphabetically, with their respective portions of the several flats collected together, set opposite to their names, and brought out in one sum ; in short giving a summary of the allotment.

TO STAKE OUT AN ALLOTMENT.

The principal station lines of the original survey being retained in the plan will materially assist in the staking out. A road, drain, or other main line, being first set out, may be used as a base, on which to found the greater part of the other work. The length of one side of a lot, will generally be determined by local circumstances, as the width of the land to which it is to be attached, or the width of the common ; if not so determined a line may be assumed. Then to find the adjoining side, divide the value to be allotted, by the length of the given side, the quotient will be the length of the adjoining side of the rectangle. If the given line, pass through several different qualities of land, as Ab, bc, cB, multiply the lengths, Ab, bc, cB, by their respective values per acre, add the products together, and divide the whole value of the lot by their sum, the quotient will be the width to be set out, if the lot be rectangular.* If not rectangular, the plan should be divided into small squares, as of chains, or perches, when the desired quantities may be readily ascertained.

The necessity for exercising the utmost care in these cases is so obvious, as to require no comment. In extensive or complicated cases, when the plan has been divided, and the lines set out on the ground, the whole should be re-surveyed, and the quantities calculated, before the fences are cut.

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* The reader who requires detailed examples of these minutiæ, is referred to "Practical Surveying," by Thomas Holliday, York.

SECTION V.

ENGINEERING SURVEYING.

To trace a New Line of Road.

THE business of tracing the line for a road, should never be undertaken without the assistance of instruments; local suggestions should be listened to, but acted on with due caution.

When it has been resolved upon that a new road shall be made, the first thing to be done is to obtain a good map of the country, lying between the extreme points. In this kingdom the maps of the Ordnance Survey are the best that can be referred to, as on them are shown not only the horizontal distances, but the general features of the country, which require to be attended to; as hills, valleys, rivers, morasses, &c. Where these, or other reasonably good maps are not to be had, a survey must be made.

It is possible to make such a survey with a chain only; measuring the angles therewith, as has been already shown.

But for a preliminary survey, the pocket sextant is the most useful instrument; after a little practice it can be used with great facility, and affords a most expeditious mode of taking angles.

A theodolite, however, is immeasurably the best instrument for this purpose. Those of five inches diameter, are the most suitable; they cost £17. or £18. and are not inconveniently heavy.

The preliminary survey should show all existing towns, villages, houses, roads, rivers, quarries, bogs, and mountains; the summits of important hills, the bottoms of deep vallies, &c.; so that, if necessary, the hills may be sketched in, and then some estimate may be formed of the course likely to prove the best.

A plan having been obtained, and a line or lines drawn on it, trial sections should be taken of such lines. These sections should all be laid down on the same datum line, by which the comparison of them will be greatly facilitated. In this preliminary survey it will not be necessary, nor even advisable, to shew the minute undulations of the surface; the leading features only, are requisite; but numerous bench-marks, say at about every mile, should be taken, on which to found or to check future operations. If the survey be extensive, a scale of ten or even twenty chains to the inch, will be sufficient for the horizontal distances; and twenty or perhaps forty feet to an inch, for the vertical scale.

If the line is an important one, and such as will justify the expense of deep cuttings, borings should be made for the purpose of ascertaining the nature of the strata, wherever deep cuttings would be required. If bogs or morasses are to be passed over, the depth of the peat should be ascertained, and the facilities or otherwise for drainage, observed. If it be necessary to cross rivers, the height of the greatest floods should be shown on the section, together with the sectional area of the river.

It may be laid down as a general rule, that the best line of road between any two points, will be that which is the shortest, the most level, and the cheapest of execution; but this rule admits of much qualification; it must in many cases be governed by the comparative cost of annual repairs, and the traffic expected to pass over the road. Natural obstructions also, such as hills, valleys, and rivers, will intervene, and frequently render it necessary to deviate from the direct course.

The general rule to be followed then, is to preserve the straight line, except when it becomes necessary to leave it to gain the desired rate of inclination, without incurring unjustifiable expense. When a deviation is made for this purpose, it becomes necessary to proceed in a direct line from a new point.

Thus, for instance, if it be decided to have no greater inclination than 1 in 35, on a new line of road, and the surveyor, when he arrives at a given point, finds a greater inclination than this, he must incline from the direct line. Then having gained the summit, he should not endeavour to get back into the original straight line, but pursue the direct line from the last gained point towards the extremity, unless he is again obliged to deviate from it. This part of the survey being accomplished, it will be necessary to examine the practicability of making a direct line between the first point, and the third, without going round to the second one.

When hills are high and numerous, it will frequently be evident, on perambulating the country, that the straight line should be departed from altogether, from the beginning, in order to cross the ridges at lower levels, by a circuitous course.

It constantly happens that although inclinations which do not exceed the prescribed limits, can be had without quitting the straight line, the ridges may be crossed at many feet less perpendicular height, by winding the road a little; but the propriety of doing so will depend upon the length by which a road will be increased by going round. The saving of perpendicular height to be passed over by a road, though a matter of so much importance, has not hitherto, in most cases, received that attention from scientific men, which it deserves. But such an investigation is more the business of the engineer, than of the surveyor, to whom therefore, it is left. Some good examples are given in Sir Henry Parnell's excellent "Treatise on Roads."

When expeditious travelling is the object, the rate of inclination that never should be exceeded in passing over hills, when practicable, is that which will afford every advantage in descending as well as in ascending; because then the time lost in ascending one hill, may be regained in descending another.

It may be taken as a general rule, in laying out a line of new road, never, if possible, to have a greater inclination than 1 in 35; as that has been found by experience, to be just such an inclination as admits of horses being driven in a stage coach with perfect safety, when descending, as fast as they can trot. The experiments detailed in Mr. Telford's seventh report of the Parliamentary Commissioners, of the Holyhead road, made with Mr. Macneill's Dynamometer,* a machine for measuring the force of traction required to draw carriages over different roads, fully establish this fact.

The results of these valuable experiments have been so frequently quoted, that it is not deemed necessary here to repeat them, especially as they concern the engineer rather than the surveyor. This latter reason also obviates the necessity for following Sir Henry Parnell through the elaborate reasoning which he adduces to prove the necessity for using moderate inclinations only.

Hilly ground is not always to be avoided, as being unfit for a road; for, if the hills are steep and short, it will often be easy to obtain good inclinations, or even a level road, by cutting down the summits, and filling in the hollows; but this must of course be regulated by the expense to be incurred.

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* Mr. Macneill has improved this instrument, by making the machinery of it register the force of traction, the inclination of the road, and the space travelled over. See appendix, No. 1, Sir H. Parnell's Treatise on Roads, for a description of the new instrument.

The great fault of most roads in hilly countries is, that having ascended to a considerable height, they descend again before they gain the summit which they have to traverse; in many instances this is repeated several times, so that the number of feet actually ascended, is many times more than would be necessary, if a height once gained were not lost again.

Sir Henry Parnell says, "An instance of this defect is observable in the line of road across the island of Anglesea, on which a horse was obliged to ascend and descend 1283 perpendicular feet more than was found necessary by Mr. Telford, when he laid out the present new line, as shewn by the annexed table:

	Heights of Summits above high water.	Total rise and fall.	Length.	
			Miles.	Yards.
Old Road	339 feet	3,540	24	428
New Road	193	2,257	21	1,596
Difference	146	1,283	2	592

The general principle to be observed in tracing a line for a road crossing a valley is, to select the highest practicable point; by which means the descent and ascent of such valley will generally be diminished.

Thus in going from one point to another, if it be found by levelling, that in a straight line the valley is too deep to make an embankment at a reasonable expense, the surveyor should try a line higher up the valley, rather than in a direction on a lower level. Instances will occur, however, where a valley may be crossed with much more advantage down stream ; as for instance, where the sides of the valley contract considerably, by reason of rock in the bottom ; less embankment may be required to raise the road to the same height, than if it were carried higher up the valley.

In either case, another reason for preferring one line to another, may present itself; which is, the facilities offered us of availing ourselves of the advantages presented by the steeper side of a tributary valley. For not only may much less earthwork be necessary, but a shorter line be practicable, and a less number of old roads be interfered with, whose levels might have to be altered, to reduce them to the level of the new road.

Sir Henry Parnell gives detailed examples of different plans, proposed by the Parliamentary Commissioners of the Holyhead road, to improve the valley of the Geese Bridge, between Towcester and Daventry; six different surveys, and plans for doing so were made, the particulars of which are given. In many situations, particularly in mountainous countries, it will be found necessary to pass valleys, or deep ravines, by means of high arches. The suspension bridge over the Menai Straits, in North Wales, is the most remarkable instance of this kind, for besides its use in passing those Straits, it has improved the road, by its being no longer necessary to descend to the level of the water, cross the dangerous ferry, and ascend again on the other side.

In some situations it may be advisable to pass through a hill by means of a tunnel, instead of deep cutting. There are three works of this kind, on the Simplon Road. One of them, " la grande galerie de Gondo," is 240 yards in length, 25 feet in width, and as many in height. There is a similar work at Puzzuoli, near Naples, which is nearly half a league long; but the construction of railways, has made tunneling an every-day matter in this country.

The mode of crossing a river to the best advantage is frequently a matter requiring much consideration ; but the same observations apply to this as to crossing valleys generally. It is not advisable, however, to cross a river at the narrowed place, unless it can be done by a single arch. A situation should be selected where the river runs in a straight course for a considerable distance above, and the channel rather narrows below ; for the piers of a bridge naturally narrow the water-way and increase the rate of the current, and may thereby endanger their own stability. Existing bridges, either above or below the site of a proposed one, should be carefully measured, and the effects of floods upon them observed, previous to determining what the water-way of the new one is to be.

It will generally be prudent to avoid laying out a line of road over bogs and marshes, if it can be done without adding too much to the length of the road. But when the additional distance would be considerable, it will be better to incur the expense of a proper drainage, and of forming an embankment to compress and harden by its weight, the moist and elastic subsoil.

It will sometimes happen, that road materials can be better obtained by carrying a line of road in one direction, than in another. This should have much influence with the surveyor, in the selection of his line.

When all other things are equal, or nearly so, the north side of a valley is to be preferred to the south, because it is better exposed to the sun. For the same reason the south side of parks, woods, &c. is preferable ; as the high fences and trees of parks, particularly when on the south side, are very injurious to road surfaces. Besides the benefit which a road receives from drying rapidly by an open exposure to the atmosphere, the free respiration afforded to horses is very advantageous to them.

Having decided upon the general direction of a road, the precise line must be selected so that the cuttings shall furnish sufficient materials to form the embankments. A great deal of expense may be avoided by a judicious management of the high and low grounds which the surface of the country affords.

The chief care, when a road must be carried over a great elevation, is, to lay it out so that it shall not have any fall from the point from which it departs, until it reaches the summit. The lowering of heights and the filling of hollows should be so adjusted, as to secure gradual and continued ascending inclinations to the highest point to be passed over.

It is very important that the longitudinal inclinations of a road should be laid out with the least possible quantity of earthwork. This can be satisfactorily accomplished only by calculating the quantities, and being careful to make the embankments balance the cuttings, so that there be no overplus to go to spoil, nor any side cuttings required.

The best and most expeditious mode of calculating earthwork, yet published, is by Mr. Macneill's "Earthwork Tables."*

Improving Old Roads.

Sir Henry Parnell observes, that "In improving old roads, nearly the same objects should be attended to as in making new ones; such for instance, as the direction, the longitudinal inclinations, the breadth, form, and hardness of the surface; the drainage, and the fencing."

There is much very valuable information on the subject of roads generally, in Sir Henry's work.†

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* Roake and Varty, Strand, London.

† It is perhaps necessary to say, that Sir Henry Parnell's language has been to a great extent adopted in the above article, because the author found the ideas ready expressed to his hand. A few practical modifications have been made in the details, but the soundness of the propositions, generally, has been confirmed by extensive personal practice.

ROAD SURVEY.

The preliminary survey having been completed, and the line determined upon by means of it, a regular survey of the selected line must be made. As none but turnpike roads will justify the expense of regular surveys, it may be assumed that these are to be prepared for Parliament, for Grand Juries, or for other public bodies; each of which bodies has promulgated rules and orders to be observed; copies of such regulations should therefore be carefully read over previously to commencing a survey.

As has been already mentioned, the best instrument to use is a theodolite. The general mode of using it, also, has been already described.

In a common-road Parliamentary survey, it is not necessary to give so much detail as for a railway, although by the "Standing Orders," it is subject to many of the same regulations. To the instructions for Railways, under the heads of "Parliamentary Plan," &c. the reader is therefore referred.

A section must also be made; the student may therefore here be introduced to the art of Levelling.

LEVELLING.

The object aimed at in levelling, is to find a line apparently parallel to the horizon, and thus to determine the relative height of one place with respect to another, that is, really, their relative distances from the centre of the earth. A line equally distant in all its parts from the centre of the earth is said to be truly level, for on such a line water would remain still, without running in either direction: and a section of ground, is a line describing a profile of the surface. A plan of a road, for instance, shows the horizontal undulations of the line; a section shows the vertical undulations; that is, the hills and hollows.

The horizontal or level line, to which the deviations are referred, is called a datum line.

On selecting a Datum.

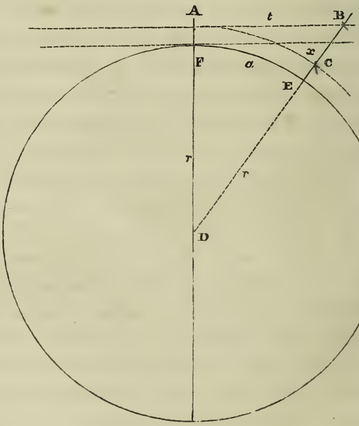
It has been usual to refer to "the level of the sea," meaning low water of spring tides, or to "high water mark," as a datum, but this is highly objectionable; it has been proved to be so indefinite as to be altogether useless, as a matter of reference. In the year 1838, Mr. Bunt of Bristol, under the direction of Professor Whewell, on account of the British Association, levelled a line across from the Bristol Channel to the English Channel, with much care; when he found that high-water mark at Portishead, on the coast of the Bristol Channel, was 13.60 feet higher than high-water mark at Axmouth on the coast of the English Channel; and that low-water mark was 12.14 higher at Axmouth than at Portishead; but that the difference of the mean level, that is the mean between high and low water of the tides of the two places, was only .73 of a foot, or rather less than 9 inches. The soundings of Portsmouth harbour, made some years ago, and referred to "Low-water," were found on a late occasion, when the plan was submitted to James Walker, Esq. to compare them with the present soundings, to be nearly useless; because the old datum was so indefinite. A fixed level, as the sill of a lock-gate, or the coping of a dock-wall, should be selected: such point being, if convenient, referred to high-water, or the mean level of the sea.

Curvature.

It is necessary that a surveyor should form a just notion of the figure of the earth, and the possible effect of that figure on some of his operations; that so he may not be led either to despise it, or to attribute an undue value to it.

The figure of the earth then, has been ascertained to be an oblate ellipsoid; the equatorial diameter exceeding the polar by 26½ miles; but as this difference is only about $\frac{1}{300}$ of the whole diameter, we may for all ordinary purposes, consider the earth a sphere or globe. The direction in which a plumb-line hangs, is observed to be, in all situations, in every part of the earth, exactly perpendicular to the surface of still water at the same point; it is also exactly perpendicular to a line truly adjusted by a spirit level.

In common parlance, when we speak of a level line, on the surface of the earth, we mean a line equidistant at all points from the earth's centre; and the earth being a sphere, such a line will of course be a curve; but the apparent level, that is the line of vision exhibited by the telescope of a truly adjusted spirit level, or as it is commonly called, an horizonal line, that is, a line parallel to the horizon (although every point has really an horizon of its own), is a right line, parallel to a tangent to the earth's surface at the



point whereon the instrument stands. Therefore, when the distance between the instrument and the observed point is great, a proportionate allowance must be made to reduce the apparent to the true level.

The effect of curvature is evidently to depress the apparent level of an object, or cause the difference of level to appear greater than it really is; thus in the figure A B is the line of apparent level, whilst A C is the true level; consequently C B is the difference; but C B is also evidently the difference between radius and the secant of the arc A C; therefore the difference between the true and the apparent level of an arc on the earth's surface, is the excess of the secant of that arc over the radius.

Putting a for the arc F E, t for the tangent A B, r for the radius D E or D F, and x for C B, the excess of the secant B C above the radius,

Then $(r + x)^2 = r^2+t^2$; Whence $a\ (2r+x)\ t^2$; but to simplify the calculation, as the line C B must always bear a very small proportion to the radius of the earth, $2r$ may be safely substituted for $2r+x$; also the length of the arc a may be taken for the tangent t; $2\,r\,x$ then becomes $= a^2$, and $x = \frac{a^2}{2r}$, which, assuming the mean diameter of the earth to be 7912 miles,* gives $x = .667$ of a foot, or 8.004 inches for one mile, taken in one right line; that is, at one sight. Now as the difference between the true and the apparent level increases as the square of the distance between the instrument and the observed object, to find the curvature for two miles, we must multiply the above deflection not by 2, but by the square of 2; it will therefore be 2.668 feet, or 32.016 inches; which for all ordinary purposes may be called $2\frac{2}{3}$ feet, or 32 inches. Consequently if t = the tangent, or distance in miles and decimals of a mile, and x = the excess of the apparent above the true level, in feet and decimals, then $x = \frac{2}{3} t^2$. This is a very near approximation when the distance does not exceed 20 miles.

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* The equatorial diameter is 7,925 miles, and the polar 7899. The equatorial circumference is 24,899.

Or, as the depression in feet $= \frac{2}{3}$ (Distance in miles)^2 $\therefore$ the depression in inches $= 12 \times \frac{2}{3}$(Distance in miles)^2 $= 12 \times \frac{2}{3} \left(\frac{\text{Distance in chains}}{80} \right)^2, = 12 \times \frac{2}{3} \left(\frac{\text{Dist. in chains}}{6400} \right)^2 = \left(\frac{\text{Dist. in chains}}{\frac{6400}{8}} \right)^2 = \left(\frac{\text{Dist. in chains}}{800} \right)^2$. Therefore divide the square of the distance in chains, by 800, the quotient will be the depression in inches.

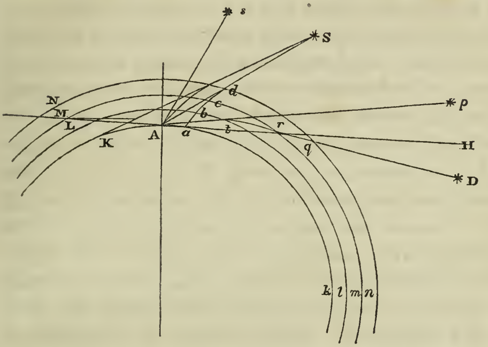
Hence it appears that the curvature for 6 chains is only .006 of an inch; consequently unless the distance of one station, from the instrument, exceeds the distance of the other, from the instrument, by 6 chains, no correction for curvature in ordinary cases is requisite.

REFRACTION.

The theoretical effect of refraction should also be understood by the surveyor. "It is the power which air possesses, in common with all transparent media, of bending the rays of light out of their straight course. Owing to this property, objects seen obliquely through it, appear otherwise situated than they would to the same spectator, had the atmosphere no existence; it thus produces a false impression respecting their places, which must be rectified by ascertaining the amount and direction of the displacement so apparently produced, before we can come at a knowledge of the true directions in which they are situated from us.

Let the object to be observed, be a star, as S, fig. 76.

Suppose a spectator placed at A, any points of the earth's surface K A *k*; and let L *l*, M *m*, N *n*, represent the successive strata or layers into which we may conceive the atmosphere to be divided, and which have spherical surfaces concentric with K *k*, the earth's surface; now if the air were away, the spectator would see the star S in the direction of the straight line A S. But in reality when the ray of light S A reaches the atmosphere, suppose at *d*, it will by the law of optics begin to bend downwards, and take a more inclined direction, as *d c*. This bending will at first be imperceptible, owing to the extreme tenuity of the uppermost strata; but as it advances downwards, the strata continually increasing in density, it will continually undergo greater and greater refraction in the same direction; and thus, instead of pursuing the straight line S *d* A, it will describe a curve S *d c b a*, continually more and more concave downwards, and will reach the earth not at A, but at a certain point *a*, nearer to S. This ray, consequently, will not reach the spectator's eye. The ray by which he will see the star, is therefore not S *d* A, but another ray which, had



there been no atmosphere, would have struck the earth at K, a point behind the spectator; but which being bent by the air, into the curve S *d c b a*, actually strikes on A. Now it is a law of optics, that an object is seen in the direction which the visual ray has at the instant of arriving at the eye, without regard to what may have been otherwise its course between the object and the eye. Hence the star S, will be seen, not in the direction A S, but in that of A *s*, a tangent to the curve S *d c b a*, at A. But because the curve described by the refracted ray is concave downwards, the tangent A *s* will lie above A S, the unrefracted ray; consequently the object S will appear more elevated above the horizon A H, when seen through the refracting atmosphere, than it would appear were there no such atmosphere. Since, however, the disposition of the strata is the same in all directions round A, the visual ray will not be made to deviate laterally, but will remain constantly in the same vertical plane, S A C′, passing through the eye, the object, and the earth's centre.

The effect of the air's refraction, then, is to raise all bodies higher above the horizon in appearance than they are in reality. Any heavenly body actually in the true horizon, will appear above it, or will have some certain altitude. Nay, even some of those actually below the horizon, and which would therefore be invisible but for the effect of refraction, are, by that effect, raised above it and brought into sight. Thus the sun, when situated at D, below the true horizon, A H, of the spectator, becomes visible to him as if it stood at *p*, by the refracted ray D *q r t* A, to which A *p* is a tangent."*

But the ordinary operations of the civil surveyor being confined to very inconsiderable distances for each observation, and those always occurring in what may be considered the same medium, that is, the same stratum of air, refraction as well as curvature may by him very safely be disregarded. This is the more fortunate, as the exact estimation of the amount of atmospheric refraction is a very difficult subject of physical enquiry, and one on which geometers are not yet agreed.*

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* Herschell's Astronomy.

It may be observed, however, that in very hot sun-shine, especially after rain, when the effect of evaporation is evident through the telescope, the accuracy of observations cannot be relied upon, and therefore the work had better be suspended until the effect has subsided.

The effects of Curvature and Refraction having been thus practically disposed of, the next thing to be considered is the adjustment of the instrument.

The use of the old "y spirit-level," was, to a great extent, superseded by "Troughton's Improved Level," and this in its turn, has been similarly set aside, by "Gravatt's Dumpy Level." To the adjustments of this last instrument, therefore, we shall confine ourselves in the present instance; and this the more readily, as the same principles are applicable and the same objects are to be attained in the adjustment of all levels.

PRACTICE OF LEVELLING.—DUMPY LEVEL.

Mr. Gravatt, by adapting a large object glass of short focal distance, to the telescope, obtains the light and power of a large instrument, without the inconvenience of its weight, &c.

A A, Fig. 77. is the telescope, having a wire-plate or diaphragm with cross wires, placed at the eye end; there are three wires, two of them vertical, between which the staff may be seen; and the third, by which the observation is made, placed horizontally. The telescope is generally

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* For the means taken by the officers, engaged on the Ordnance Survey, to counteract this cause of error, see "Trigonometrical Survey," 175, and following pages, or outline, &c. of "Trigonometrical Survey," by Lieut. Frome, R. E. p. 73, &c.

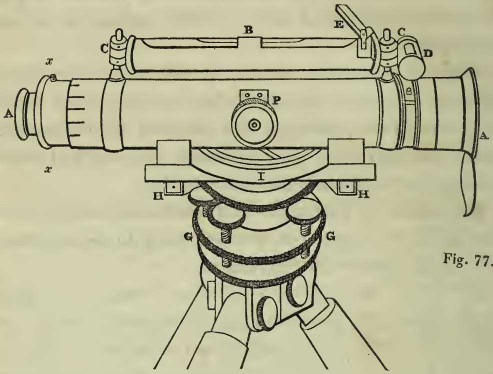


Fig. 77.

constructed to show objects inverted; and as such a telescope requires fewer glasses than one which shows objects erect, it has the advantage in point of brilliancy; and when an observer has become accustomed to it, the apparent inversion will make no difference to him; he must be exceedingly careful, however, if he should occasionally be led to use an instrument which does not invert. A diagonal eye-piece sometimes accompanies the instrument, by which objects are seen in their natural position; it is advisable to use it always, or not at all. A cap is adapted to the object-end of the telescope, to screen the glass from the rays of the sun, or from rain.

The requisite adjustments are, that the axes, or line of collimation of the telescope and of the level, be parallel to each other; and that the telescope be at right angles to the vertical axis, that is, that the air-bubble, having been brought to the centre of the tube by means of the parallel-plate screws, preserve its position while it is turned round horizontally on the staff-head. The adjustment of the level is effected by correcting half the observed error by the screws H H, and the other half by the parallel-plate screws G G; the covers of the screws H H may be taken off during the adjustment of the instrument.

It will be perceived that there are three screws at H, the two outermost of which pass through the bar, the centre one presses against it only; therefore to raise one end, let the outer screws at that end, be slacked, and the centre one be tightened. If the horizontalism of the level is much out of adjustment, it ought to be corrected previous to the collimation; it may, in that case, be done by the screws at C C, but it is evident, that if these screws be disturbed, they will destroy the collimation, which would consequently require re-adjustment; so that if the instrument has been corrected for collimation, the screws H H only, should be moved for the other adjustment.

For Collimation.

Having set up the instrument on a tolerably level piece of ground, as at Y, fig 78, and the eye-piece A being drawn out, so that the wires are seen distinctly and well defined, move the object glass by the screw P, until distinct vision is obtained without parallax; which will be the case if, on looking through the telescope at some distant object, it and the horizontal wire remain steadily in coincidence; if the wire has any parallax, it will appear unsettled and wavering. Move the telescope till it stands in the direction of two of the parallel plate-screws, and by turning those screws bring the air-bubble to the middle of the tube, as shown by the scratches on the glass.

Now let two pegs be driven into the ground, about six chains apart; that is, three chains each way from the instrument; let a staff be held on each peg, and the readings of both staves be noted, the bubble being kept in the centre of its run by means of the parallel plate-screws; then, although the instrument be out of adjustment, yet the points read off will be on the same level. For it has been already shown, that in such inconsiderable distances, both curvature and refraction may be disregarded; and as the distances are equal, the errors of the instrument will counterbalance each other. Thus suppose the error to be upwards, as to A and B in the figure, it is clear that the quantity of error, for the same distance, will be the same both ways.

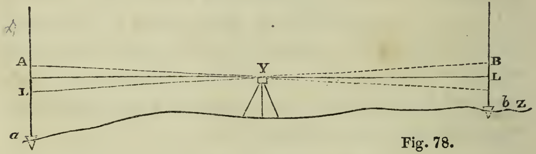


Fig. 78.

Now remove the instrument to a point, as Z, a short distance beyond one of the pegs, as B; and having levelled it, read both staves, that is, get a reading from each peg; now, if the collimation of the instrument be correct, the difference of the readings will be the same as before; thus, suppose the staff A, from Y reads 5 feet, and B 3 feet; and from Z, A reads 6 feet, and B 4 feet, the collimation is correct, because the difference, viz. 2 feet, is the same from both stations. But if the difference is not the same, the instrument must be adjusted, by altering the screws *x x*, which adjust the diaphragm and consequently the wire or horizontal spider line, until the difference of reading is the same from Z and from Y. This will most probably require frequent trials. One of the screws should always be loosened, before the other is tightened.

If a piece of still water, as a pond, or canal, be at hand, it will furnish the simplest and most convenient means of adjusting a level, on a calm day. Let two pegs be driven in about six chains apart, exactly to the surface of the water when it is undisturbed; set up the instrument, and level it, within six or eight feet of one of the pegs, and having a staff on that peg, note its reading, then remove the staff to the other peg, at which the reading should be the same as on the first; if not the collimation must be corrected, as already directed.

At D, fig. 77, is represented a small level placed across the telescope at right angles to the principal level C C; it may assist in setting up the instrument approximately level, by means of the legs.

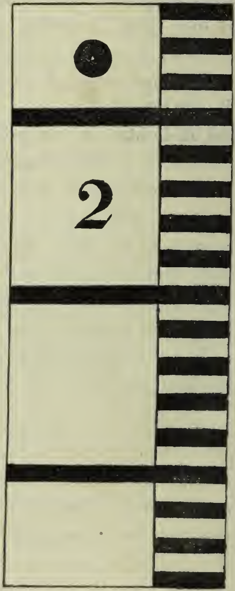
A mirror, mounted on a hinge joint, is represented at E; its intended use is to reflect the image of one end of the air-bubble, in the principal level, to the eye, so that the observer, having carefully adjusted his instrument, can whilst reading the staff, see that the bubble retains its position; this may be found useful on boggy or elastic ground, or in windy weather. The observer must keep both eyes open, one on the mirror, and the other, through the telescope, on the staff. The parallel plate-screws, are similar to those of the theodolite, and of other levels.

At I is represented a very ornamental compass, which has been adapted to this instrument.

LEVELLING STAVES.

The staff has gone through exactly the same gradations as the level. The old staff was superseded, in a great measure, by "Troughton's levelling-staff," and this again by "Gravatt's Staff." These last have no vane to slide up and down, but the face of the staff is made broad enough to contain sufficiently large graduations and figures for the observer to read at the distance of 10 or 12 chains; thus securing greater certainty and expedition in the progress of the work. The inconvenience and uncertainty experienced in using the vane staff, in extensive operations, are very great, so much depends upon the staff-holder; but with this new staff, the surveyor is to a great degree independent of his assistant, so much so, that any steady labouring man may be employed in that capacity. For very nice operations, however, some surveyors will continue to use a vane staff.

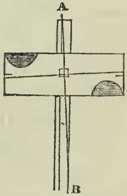
The newly constructed one consists of three parts, which pack neatly together, and when connected for use, form a staff seventeen feet in length. This staff is divided into hundredths of a foot, coloured alternately black and white, and occupying half the width of the staff; but, for distinctness, the lines denoting tenths of feet, are continued throughout the whole width, as in fig. 79, each foot is numbered; and there is a large dot at each side of the five-tenths division.



Several attempts have been made at improvements in this staff, but their success is very problematical. Mr. Sopwith, for instance, has introduced one in which distinctive figures are attached to every other hundredth of a foot; the mechanical construction also differs from this, it is more elaborate, and consequently makes the staff more expensive. It is very neat, however, but is subject to injury in windy weather. Another gentleman claims much merit for having inverted the figures, to counteract the inversion of the telescope; but it must be borne in mind, that the scale remains apparently inverted; so that instead of proceeding naturally, from the upper side, or top of the figure, forward, for the additional decimal parts, the observer must go backwards, as it were, to reckon forward, from the bottom of each figure. Tripods, and feet, and shoes too, to place the staff upon, have been invented by some, but the most convenient way, for Parliamentary or other temporary purposes, is to have the staff set upon a stone, or other hard substance, which can almost always be found by a little attention. For permanent works, pegs should always be driven into the ground.

Levelling staff used by John A. Lloyd, Esq. F. R. S. in ascertaining the difference of level, between the River Thames at London Bridge and the Sea at Sheerness, in Feb. 1830.

Major Lloyd having given a minute description and drawing of the level,* proceeds to say, " Being now in possession of an instrument, equal to perform the most delicate observations, my next object was to make some improvement in the station-staves, so that they might point out as minute a quantity as the instrument could detect a difference of."



The staff itself is a rod of six feet six inches in length, of solid seasoned mahogany, on the face of which is let in a slip of brass, rivetted at intervals from end to end; by the side of this is also a slip of holly, fixed in the same manner. The divisions are laid down in feet and tenths, on the holly; and in feet, tenths and hundredths, on the brass. The lower part of the staff is fitted into a square tube of brass, about eight inches long, four inches of which are occupied by the staff, and the remainder filled up with lead.

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* Vide Philosophical Transactions, for 1831.

The vane is a plane of seasoned holly, with two semicircles of stained ivory, let into the face of it. It is fixed by screws to a brass box, with tightening springs on the staff itself; there is another small slide on the staff, having two clamping screws, and a long tangent screw, which is attached to the box of the vane by a female screw; the last gives a slow vertical motion to the vane.

On the top of this vane, and at right angles to one another, are two small spirit levels, mounted in brass; affixed to the vane, in a square hole in the centre, and levelled on one side, is a small brass vernier, the edge of which slides on the divisions of the staff reading to the 1000dth, and by practice, to the 10,000dth of a foot.

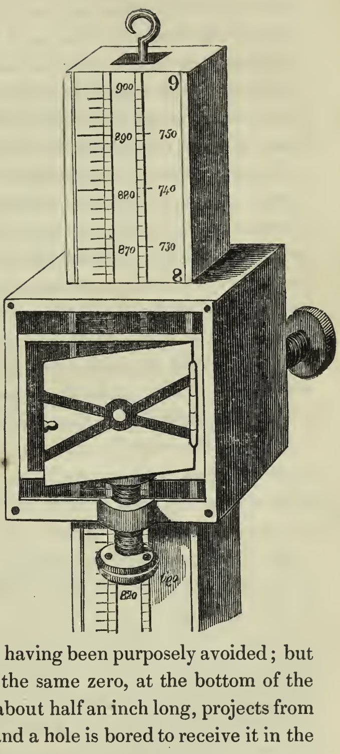
The station-staff, thus described, is in itself complete; but for accurate observation it requires to be immoveable on the picket: a three-legged support is therefore added, having a box and ring with double compass gimbals, and a horizontal motion; into this the staff slides.

There is also a brass tripod, with iron legs, having a hole in the centre plate, over which another small plate slides, fixing to the tripod by two clamping screws; this is used to confine the bottom of the station-staff over the picket.

Major Lloyd also describes the mode of adjusting this staff vertically over the picket.

Mr. Bunt's Staff.

Mr. Bunt. of Bristol, has in the "Transactions of the British Association, Vol. VII." given a drawing, and the



following description of the staff he made use of. He says, "A brass levelling staff was employed in levelling between Bristol and Portishead; but being found inconvenient, and liable to get out of repair, was laid aside. The staff subsequently used, (fig. 80), was a single piece of straight grained oak 9 feet long, and 2 inches wide. On the face are two different scales of equal parts One of feet and hundredths of a foot; the other has larger divisions, in the proportion of 19 to 16, nearly, or more exactly, as 1.18702 to 1; an aliquot ratio of the scales having been purposely avoided; but both are reckoned from the same zero, at the bottom of the staff." "A stud of wire, about half an inch long, projects from the bottom of the staff, and a hole is bored to receive it in the top of the peg, which is driven into the ground at every station, and on which the staff rests during the observation. A small spirit cup, with a glass cover, screwed to the lower part of the staff, serves to adjust it to a vertical position, in which it is held fast by a clamp attached to three strong legs, jointed and folding together, in the usual manner."

"The vane is a small mahogany box, about 3 inches in each dimension, open at the ends to admit the staff. Two large wooden screws at the back of the vane clamp it very firmly to the staff. In front is a frame of brass, about 2 inches square, sliding within an outer frame of brass screwed to the vane, with a range of motion of about half an inch, up or down, moved by a large vertical screw, with a milled head, working through the lower part of the outer frame. A square aperture, corresponding with the inside of this frame, is cut through the mahogany, in order that the divisions of the staff may be seen. A small ivory door, moving on a hinge, is fitted into the sliding frame, on which are drawn two lines," as shewn in the drawing. "Attached to the inside of the sliding frame, and exactly behind the centre of the white circle, is a vernier, nearly in contact with the face of the staff, which divides the hundredth of a foot into five parts, of 20 ten-thousandths each, so that the observation is read off to four decimal places."

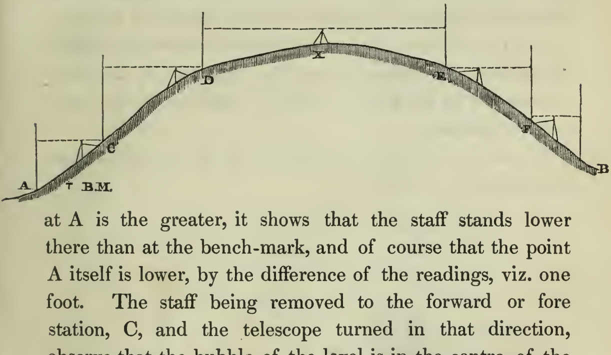
"When the vane was raised so near the top of the staff, as to be out of reach of the hand, the adjusting screw was worked by a long fork of stout wires, thrust into holes, in the milled head, made to receive it."

Mr. Bunt proceeded as regularly as possible, by stations of four chains, driving a peg at each, and attained considerable accuracy.

Where great minuteness is required, a double scale is perhaps advisable, but there is an air of mystery about Mr. Bunt's odd scale, the necessity for which is not so obvious; one of inches and tenths would be simpler, and answer the purpose quite as well.

Major Lloyd's staff was altogether too short for getting through business; indeed he complains that the levelling up the hill to Greenwich Observatory, some seventy or eighty feet, was a very laborious operation.

The young surveyor then being provided with a level properly in adjustment, and either one, or two staves, as the case may be; that is, if expedition be the principal object, two staves should be used, if minute accuracy, only one; a proper bench-mark having been fixed upon, let the staff be held perfectly upright on it; set up the level in the direction in which you are going to proceed; it is not by any means necessary to be exactly in the line, but in a convenient position to command the bench-mark and as much of the line as possible: having set down, in its proper column in the field book, the reading of the staff on the bench-mark, let the staff be placed on the commencement of the line, as at A, fig. 81; suppose the reading on the bench-mark to have been eight feet, and on the station A, nine feet, the difference is one foot; and as the reading at A is the greater, it shows that the staff stands lower there than at the bench-mark, and of course that the point A itself is lower, by the difference of the readings, viz. one foot. The staff being removed to the forward or fore station, C, and the telescope turned in that direction, observe that the bubble of the level is in the centre of the tube, if not, touch a pair of the parallel plate screws* so as



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* It is manifest that on principle, the parallel plate screws should not be moved after the level has been once adjusted at a given station, but in practice the instrument is very frequently a little out of adjustment, so that it is impossible to get on without acting on those screws; it should be resorted to as little as possible however; but the student must be careful never to set down a reading unless the instrument is level, as otherwise he may have observed a point considerably higher or lower than that of the true level.

to bring it back, and then read the staff; say the reading is 0.5 of a foot, deduct this from the reading of the staff at A, viz. nine feet, that having now become the back-station, and the difference, 8.5 feet, is evidently the difference of level between the points A and C. Remove the instrument to a convenient position for observing the staff at C and D, and proceed as before, being especially careful that the staff is held in precisely the same spot as before. C now becomes the back station, and must be entered in its appropriate column accordingly. Having in this manner obtained a connected series of observations from A to B we obtain the relative heights of those places; for having ascertained the difference of height between A and C, and then between C and D, it is clear that by adding those differences together we shall have the difference between A and D, and so of the others, until we arrive at E, when the descent commences; then the readings of the fore-stations will be the greater, and the readings of the back must be deducted.

Let the following be the readings of the several stations represented in the figure.

	Back-sights.	Fore-sights.	
	8.00	9.00	A
	9.00	0.50	C
	12.10	1.20	D
	7.00	6.00	E
	0.40	8.50	F
	0.75	7.50	B
	37.25	32.70	
Fore-sights	32.70		
Difference	4.55		

Hence it appears that the sum of the readings of the back stations exceeds the sum of the readings of the fore by 4.55 feet, and consequently that the bench-mark is so much lower than the point B.

If a section were required the proceeding with the level on the ground would be nearly the same, except that on arriving at the summit x, on which the instrument stands, its height should be noted as a station, if in the line, as in the following examples, if not, the staff should be held in the line, opposite to it. The horizontal distances must be measured, and the difference of height of each pair of stations set down in their appropriate columns, as follows, viz.

FIELD BOOK.

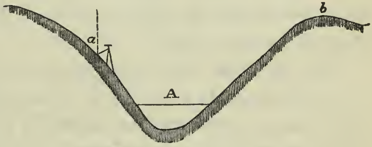
Rise.	Back sights.	Fore sights.	Fall.	Reduced levels.	Distances chains	Observations.
				100.00	· · :	B.M. top of post
	8.00	9.00	1.00	99.00	000	A ace.
8.50	9.00	0.50		107.50	220	C
10.90	12.10	1.20		118.40	560	D
2.50	7.00	4.50		120.90	970	x (Instrument)
	4.50	6.00	1.50	119.40	1380	E
	0.40	8.50	8.10	111.30	1670	F
	0.75	7.50	6.75	104.55	1840	B
21.90	41.75	37.20	17.35	100.00		
17.35	37.20					
4.55	4.55			4.55		

The Back-sights and Fore-sights having been set down in their respective columns, the greater quantity deducted from the less, for each station, and the differences set on the same line with them in their appropriate columns, the four columns should be separately added up; and if, as in the example, the sum of the rises exceeds the sum of the falls, deduct the falls from the rises, that is, deduct the less from the greater; also deduct the sum of the fore-sights from the sum of the back, that is, deduct the less from the greater of these sums also, when, if the several subtractions and additions have been done correctly, the two differences will be alike, as in the example; where it is 4.55 feet.

If those differences are not alike, the work must be examined before any further step is taken. This preliminary operation being correct, we may proceed to reduce the levels into one column for plotting.

In the example it will be perceived that 100 feet have been set down to begin with; this became advisable, indeed necessary, in this form of field-book, because there is a fall at the first station, therefore, the relative height of A could not be expressed unless a negative sign were used, which is practically very inconvenient. The point on which the staff is first held, is the zero to which all the subsequent stations refer; but this point may for convenience be called anything, as 10, 100, 500 or 1000 feet, and then to ascertain the relative height between this datum point, and any station, let the "Reduced level" of the station and this assumed datum height, be deducted one from the other, the less from the greater; and the difference will be the difference of height required. Thus, deduct 99.00 from 100.00, and the difference, viz. one foot, is the difference of level, as already shown, between the bench-mark and the station A. Deduct 100.00 from 120.90, and the difference, viz. 20.90 feet, is the difference of height between the bench-mark and the summit; the station which is represented by the higher number, always being so much higher than the other station, as that number exceeds the one which represents the other station. If the difference of height were required between any two of the stations in the line, as between A and X, the height of A, viz. 99.00 should be deducted from the height of X, viz. 120.90, by which it appears that X is 21.90 feet higher than A. The difference between the assumed height, and the last height in the page, must be the same as the differences already found in the other columns; in this case 4.55 feet. If the starting point is the lowest in the section, then the natural numbers may be used without any addition, but if any other point falls below it, such a height should be assumed as will keep all above the datum or horizontal line. It is evident that where a series of levels occupies more than one page, the last “ Reduced level” in a page should be carried forward, and placed at the head of the column of “ Reduced levels” in the succeeding page, and so the series be continued to the end; the difference between the number so brought forward and that at the foot of the same page, denoting the rise or fall in that page. Each day’s work should be closed at some well defined point, as, the centre of a road, the bank of a brook, &c. and a good bench-mark be taken, that so the work may be resumed without difficulty. If either of these points be neglected much inconvenience and uncertainty may ensue.

The series of observations must not be broken; for instance, suppose it were necessary to cross a river, as A, having very steep banks, and that the instrument has been set up at the point represented in the figure, for the purpose of ascertaining the height of the surface of the water, but the surveyor wishing to place his staff advantageously for getting on; and the difference between the distances, from *a* to the instrument, and from thence to *b*, being too great to admit of his setting up the staff at *b*, he has it removed back to *a*, and then proceeds with his level to or near *b*, from whence he is enabled to take a long fore-sight; but to accomplish this, he must work his book back to *a*; say the reading at *a* was one foot, and at the surface of the water 14.0 feet, then 1.0 would be the back-sight, and 14.0 the fore-sight; but to remove the staff back to *a* these



must be reversed, and 14.0 be placed in the column of back-sights, and 1.0 in the fore-sight column; that so the station next to be observed, shall have been the last set down, and thus the continuity of the series be preserved.

In a permanent section, where it is necessary to show the width of water surface, its height should be repeated in the field book, thus

Back-sights.	Fore-sights.	Distances.	Remarks.
2.00	1 00	6000	Point *a*
1.00	14.00	6115	River surface
14.00	14.00	6315	Do. width
14.00	1.00	Out.	Point *a*
13.00	5.00	6660	Point *b*

By keeping the field book in this manner, the work may be plotted at all times, without any effort of memory.

The word "Out" being written, instead of a distance, denotes that a point has been observed, which is not to be plotted, as part of the section; all bench-marks should be entered in the same manner, "B. M." signifies bench-mark, and should be written in the "Remarks" column opposite the word "Out."

Some surveyors reduce their levels in the field, but it is not a commendable practice; there is plenty to occupy a man's attention without that; indeed if he uses two staves it is an unjustifiable waste of time, for he consumes that of all his men, as well as his own. With one staff, in a flat country, it may be done, but not otherwise.

The field-book may be most conveniently kept with a good drawing pencil, of medium hardness.

It is advisable for a young surveyor always to carry on his field-book, in the direction which will admit of his plotting it from left to right; thus if a section were to be made from London to Birmingham, the work should be commenced at London, and all the work be done towards Birmingham. This however is not always convenient, for suppose two surveyors undertake the work, and one starts at London, whilst the other starts at Birmingham, and they work on until they meet; in such a case, to make a continuous section, one must commence his plotting at his left hand, and work towards the right, and the other commence at his right hand, and plot towards his left, otherwise the sections cannot be connected; because the extremities, instead of being one on the right hand, and the other on the left, will be both on the left. This will not present any difficulty after a little practice, but it appears more natural to a beginner, to plot from his left hand.

Bench-Marks.

Remarkable or well defined objects, should always be selected as bench-marks, that so they may be readily found by other surveyors, from the description in the field-book. Thus, "B. M. top step at Mr. Fizarkerley's front door;" or "B. M. top of 50th mile-stone;" or "B. M. lower staple of hanging post of gate, commencement of line;" or "B. M. Key-stone, East side of Gloster Bridge," &c.

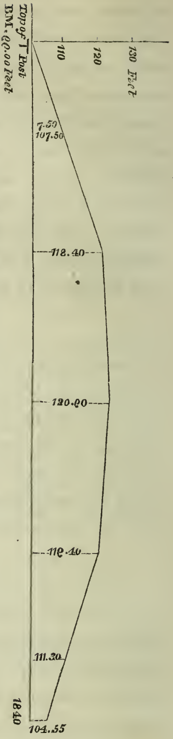
Section.

It is hoped that sufficient has been said to enable the student to comprehend the field-work. To plot the preceding field-book, let a fine horizontal line be drawn, to represent the datum, and on that line mark the several distances. The vertical scale is always considerably greater than the horizontal one; because if a scale sufficiently large to show the vertical heights distinctly, were used for the horizontal distances, these latter would run to a most inconvenient length on paper; and if a scale adequate to represent the horizontal distances, were applied to the heights, the undulations would be frequently almost imperceptible.

When a plan and section are made of the same line, they should be both laid down on the same horizontal scale, for facility of reference.

The scale will depend upon the nature of the work. Thus a trial section may be laid down at forty or fifty chains to the inch, for the horizontals; and 40, 60 or even 80 feet to an inch for the verticals. A Parliamentary section, to a scale of 20, 40 or 50 chains to the inch, and 40, 80 or 100 feet to the inch for the verticals. The best scale for a working section is, either two, or four chains, to an inch horizontally, and 20 feet to an inch vertically.

The scales having been decided on, and the horizontal distances, marked on the datum line, perpendiculars must be raised at those several points; this may be accomplished by erecting perpendiculars carefully at considerable intervals, as at 60 or 80 chains, and drawing the intermediate ones parallel to them. * The respective heights, then, being marked on these perpendiculars, a line connecting those points, will form the required section.



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* Section paper, first introduced by Mr. Brunel, I believe, may be used. It is produced by a plate, well divided and engraved in squares, each square inch being divided into 20 parts each way; so that to plot a section to a scale of 20 chains horizontal, and 20 feet for the verticals, one horizontal division represents one chain, and one vertical division one foot. The paper may, of course, be used for any multiples or sub-multiples of those scales.

It will be perceived that although, for convenience of reference, the datum line has, in the field-book, been called 100 feet, yet in the section, for convenience of plotting, it has been made use of as if still 0: this is altogether an arbitrary arrangement, and may be variously modified according to circumstances.

It is advisable to draw pencil lines parallel to the datum, at every 10 or 20 feet, according to the scale; it facilitates the work, and prevents errors.

Some surveyors insert every height in figures, as in the example, which, in a rough country, is very tedious, and, generally speaking, unnecessary.

The "Standing Orders" of Parliament, which apply to Turnpike Road Bills, are embodied in those relating to Railways, to which the reader is therefore referred.

Permanent Line.

The duties of a Surveyor, after an act of Parliament has been obtained, will be, to a certain extent, similar to those for a Railway. To the instructions under the head of "Permanent line," for Railways, he is consequently referred.

Railways.

Selection of a Line.

The same general principles apply to the selection of a line for a Railway, as for a Common Road; but as the former require so much better gradients than the latter, the selection is a matter of far greater consequence, as it involves such heavy expenses. Much care should therefore be bestowed upon observing the general formation of a district, previously to attempting to trace a line through it. The fall of water is the grand guide. From the physical fact of water seeking what is commonly called its lowest level, and that by the shortest line, or the line of greatest declivity, it follows, that the water courses mark the lowest points of the valleys, and are their lowest lines of greatest declivity. If then on a map of any portion of a country, it be found that the water courses all diverge from, or converge towards one point, it will indicate, that the first is the highest, and the last the lowest point, of that portion of the country.

If two water-courses flow in opposite directions from a point, it will indicate that it is the lowest point of the ridge of the rising ground, by which their valleys are separated; for the ridge will most probably decline on both sides, to this point.

The meeting of the waters of two streams, which have been running in different directions, indicates the lowest point, in that parallel, in the immediate vicinity.

If two water-courses flow in the same direction, and parallel to each other, at opposite sides of a ridge, the course of their currents will indicate the general inclination of the country. If two streams having run parallel to each other to a certain point, suddenly assume divergent directions, it will indicate that this is the lowest point of the ridge between them.

Several trial sections should be taken; for although they are somewhat expensive, yet the after expenses, and success of a line, depend so very much on this preliminary operation, that the money laid out on trial sections, is well expended. Of course sound judgment is requisite to enable a man, even with numerous sections, to decide upon the best line. It would not be difficult to point out instances where excessively expensive errors have been committed; such as making tunnels, and even inclined planes, where they could very easily have been avoided; taking the wrong side of a valley, and so increasing the earthwork, &c. &c. thus burthening the undertaking, first, with the expense of executing such works, and ever after with the expense and inconvenience of maintaining and working them.

When it has been resolved upon to construct a Railway in a given direction, a map of the district should be carefully examined, and certain fixed points determined; such as towns, harbours, &c. A line connecting these points, having been drawn in pencil on the map, a section should be run as nearly as possible in that line, but avoiding all palpable obstructions. Well defined bench-marks should be left at stated intervals, say at every mile. A competent surveyor being in possession of such a section, will be enabled to lay down the gradients in a general manner, and thus to ascertain whether or not a Railway can be constructed in that direction, within practicable limits; and then to observe which portions most require amendment. Trial sections of such defective portions should then be run; referred to the former bench-marks, and laid down on the first section, that so they may be readily compared.

As in the case stated for Common Roads, it will sometimes be advisable to deviate altogether from the straight line, from the beginning. A main point to be settled as soon as possible is, the most judicious manner of crossing valleys and ridges. Generally speaking, the narrowest places are to be preferred, having especial regard, however, to the tributary valleys, and to the general nature of the country, as to morasses, mountains, &c.

It is sometimes necessary that two lines of Railway should cross each other, on the same level. Such a crossing should, if possible, be arranged to occur on an embankment of moderate height, that so no obstruction may impede the sight of the conductors of trains. If this cannot be accomplished, the crossing in a curve in a deep cutting should be avoided at whatever expense. Indeed all curves in cuttings are highly objectionable; they have already led to several destructive collisions. But the engineer who, considering the present state of knowledge on the subject, arranges a crossing of two Railways, on the same level, in a deep cutting, and in a curve, ought to be tried for manslaughter, on the death of the first man which occurred in consequence of such mal-arrangement. If done through incompetence, he should be taught that incompetent persons are not to thrust themselves, with impunity, into positions involving such fearful responsibility. If done in defiance of adequate warning, a judge might feel himself called upon to inflict the heaviest punishment allowed by law.

PARLIAMENTARY PLAN.

The general direction having been determined upon, a Survey must be made of the line selected. By the "Standing Orders" of Parliament, it is required, "That a plan, and also a duplicate of such plan, on a scale of not less than four inches to a mile, be deposited for public inspection," &c. "which plan shall describe the line or situation of the whole of the work, and the lands in, or through which it is to be made, maintained, varied, extended or enlarged, or through which every communication to or from the work shall be made," &c. and

"That where it is the intention of the parties to apply for powers to make any lateral deviation from the line of the proposed work, the limits of such deviation shall be defined upon the plan, (see lines x x, z z, fig. 1, pl. III:) and all lands included within such limits shall be marked thereon, and that in all cases, an additional plan of any building, yard, court-yard, or land within the curtilage of any building, or of any ground cultivated as a garden, either on the original line, or included within the limits of the said deviation, shall be laid down on the said plan, or on the additional plan deposited therewith, upon a scale of not less than a quarter of an inch to every 100 feet."*

It has been usual to apply for power, to deviate to the extent of 100 yards on either side of the line; but as the line cannot be accurately laid down until after the survey has been made, it is frequently necessary to embrace within the limits of the survey, a space considerably greater than five chains on each side. The extent of survey required will depend upon the nature of the country, and of the obstacles; but it should never be less than five chains, and seldom more than ten chains, on each side of the line. When the centre line has been laid down, the boundary lines may be drawn as may be considered advisable. It is well always to take plenty of room, but a particular owner's land may sometimes be altogether omitted, and so his opposition to the bill avoided, by judiciously drawing the boundary lines.

SECTIO-PLANOGRAPHY.

The "Standing Orders" (for 1841) require, "That in all cases where it is proposed to make, vary, extend, or enlarge any Railway, the plan shall exhibit thereon the height of the several embankments, and the depth of the several cuttings respectively, on a scale specified thereon."† This has been denominated Sectio-Planography, and was introduced into the Standing Orders, in consequence of evidence given by Mr. Macneill, before the Select Committee of the House of Commons, on the Standing Orders, on Private Bills, June 27, 1837.

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* When the scale of a plan does not exceed six inches to a mile, offsets of less than 20 links cannot be shewn on the paper, and therefore need not be taken on the ground; but at five chains to an inch offsets of two or three links may be shewn.

† See the dotted lines on figures 1 and 5, pl. III.

Mr. Macneill, in a letter on the subject, observes that, "The strong line on these plans represents, as usual, the position of the proposed Railway line, upon the ground; but in addition, I consider it to represent a vertical section of the rail, and then I exhibit the relations of the rail-line, to the existing surface, by laying down a vertical section of the ground, keeping the surface line above or below the Railway line, according as it rises above or falls below the latter, precisely as is done in the usual detached section." Mr. Macneill would colour the parts which are found above the Railway-line red, and those below it blue; but this colouring is not required by Parliament. In a line illustrated in this manner, the character of the surface in the direction of the Railway, is at once evident; its elevations, depressions, and levels, at whatever height, are instantly seen, and in their exact positions. If cuttings, or tunnellings, be necessary, the red tint marks their situation and extent, and any embankments which may be required, are as accurately depicted by the blue. No measurement or estimate is necessary, to determine the position of a required point on the plan corresponding to any given point on the section, and *vice versâ*. They are here as they are in nature, in intimate union, and their peculiar combination or relation, can be instantly and accurately decided upon.

Mr. Macneill proposes, that when plans are lithographed or engraved, to avoid the delay that might arise from colouring the cuttings and embankments, that the former should be denoted by perpendicular lines, and the latter by horizontal ones, and that in such parts as have neither, but the line running along the natural surface, neither colouring, nor lines of any kind should be introduced. This, however, is not required by the Standing Orders; nor does it appear to be necessary; because a man must exercise his brains sufficiently to enable him to understand, that the surface represented above the line of rails is intended to be reduced to that level, and that the surface below the line is to be raised to that same level.

By the Standing Orders, it is directed "That in all Railway Bills there be inserted the following clause, (among others,)

"And be it further enacted, that it shall not be lawful to diminish the radius of any curve, as described on the plan deposited with the Clerk of the Peace, or principal Sheriff-Clerk, unless such radius exceed one mile, nor to diminish it in any such case, so that it shall become less than one mile, nor to diminish any greater radius by more than a quarter of a mile, unless where it exceeds two miles, or by more than half a mile, unless where it exceeds three miles on the said plan."

It is also enacted, that "a copy of so much of the said Plans and Sections, as relates to each parish, in or through which the work is to be made, maintained, varied, extended or enlarged, (see fig. 5.) together with a book of reference thereto, shall be deposited with the parish clerk of each such parish in England, the schoolmaster in each such parish in Scotland, (or in Royal Burghs, with the Town Clerk), and of the post-master of the post-town, in or nearest to such parish in Ireland."*

PARLIAMENTARY SECTION.

By the "Standing Orders" (for 1841) it is required, "That the section for a Turnpike Road, Cut, Canal,

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* The time for depositing plans, sections, and books of reference has not been stated above, as that and the necessary notices require the attention of a solicitor. The getting up of the book of reference, should also be done by a solicitor.

Reservoir, Aqueduct, or Railway, shall be drawn to the same horizontal scale as the plan, and to a vertical scale of not less than one inch to every 100 feet, and shall show the surface of the ground marked on the plan, and the intended level of the proposed work, and a datum horizontal line, which shall be the same throughout the whole length of the work, or any branch thereof respectively, and shall be referred to some fixed point stated in writing on the section, near either of the termini. (See line D. D. fig. 2, pl. III.)

" That in every section of a Railway, the line marked thereon, shall correspond with the upper surface of the rails. (See line S. S. fig. 2.)

"That a vertical measure from the datum line, to the line of the Railway, shall be marked in feet and inches, at every change of the gradient or inclination (see E. F. fig. 2); and that the proportion or rate of inclination between each such change shall also be marked. (See G. G. fig. 2.)

" That the height of the Railway over or under the surface of every turnpike road, navigable river, canal or railway, or junction with a railway, shall be marked in figures at every crossing thereof (see I. I. I. fig. 2); and if there shall be no such crossing in every half mile, then the height over or under the surface of the ground shall be marked once in that space (see O. O. O. fig. 2); and if any alteration in the present level or rate of inclination of any turnpike road, carriage road, or railway, be intended, then the same shall be stated on the said section. (See P. fig. 2.)

" That where tunnelling as a substitute for open cutting, or arching as a substitute for solid embankment, be intended, the same shall be marked both on the plan and section. (See Q. R. figs. 1 and 2.)

"That parties desiring to deviate from the line of any Railway, of which the plans, sections and books of reference shall have been deposited, and for which the notices shall have been given as before directed, shall be permitted so to do, provided no one deviation shall exceed one mile in length," &c.*

It is directed also "that in all Railway Bills there be inserted the following clauses:"

"That it shall not be lawful for the said company to deviate from the levels of the said Railway as referred to the common datum line described on the section so approved of by Parliament, and as marked on the same, to any extent exceeding in any place five feet, or in passing through towns two feet, without the consent of the owners, lessees, and occupiers of the land in, through, or over which such deviation is intended to be made; or in case any street or public carriage road shall be affected by such deviation, then the same shall not be made without the consent of the trustees or commissioners, or, if there be no such trustees or commissioners, without the consent of two or more justices of the peace in petty sessions assembled for that purpose, and acting for the district in which such street, or public carriage road may be situate, or without the consent of the commissioners for any public sewers, or the proprietors of any canal or navigation affected by such deviation; and that no increase in the inclination or gradients of the said Railway, as denoted by the said section, shall be made in any place to an extent exceeding the rate of three feet per mile," &c.

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* This clause refers to plans lodged in March, but by a resolution of the House of Commons, passed in the session of 1842, plans are not required to be lodged until the 30th November immediately preceding the session in which application for the Bill shall be made.

PERMANENT LINE.

Having obtained an Act of Parliament it will be necessary to re-survey the whole line. For this purpose, the best way will be to measure a base line, as nearly as possible in the Parliamentary line, and to mark such base by substantial pegs at regular intervals, the length of such intervals being regulated by the nature of the country; in a hilly country they should not exceed five chains; in a flat country, where the line would probably be straight, they may be of twenty chains; but, generally speaking, distances of ten chains each will be the least liable to error. The base having been measured, and the angles carefully taken with a theodolite, should be laid down on paper.

A section should then be taken of the base line, showing the principal undulations of the surface, the height on every peg, the crossings of roads, &c. &c. The gradients, or line of railway, being laid down on this section, the surveyor will be furnished with sufficient data to enable him to set out the line; for let him take a tracing of the section, and on it mark the amount of cutting or embankment on each peg, then, returning with his tracing to the ground, he will be enabled to mark the level line, that is, the line of the surface of the rails, at right angles to each peg.

If it be deemed advisable to lay down the necessary curves on paper previous to setting them out on the ground, transverse, or cross sections may be taken from these level pegs to the adjacent base pegs, these together with the positions of the pegs being marked on the plan will furnish good data for accomplishing this object.

But in a hilly country, especially where the hills are steep, the most expeditious and satisfactory mode is, for a competent surveyor to provide himself with a dozen or more neat, perfectly straight, and well made, ranging poles; in a very rough, or wooded district, these should be eight or ten feet in length; in a flat district they may be much shorter; and also with an offset staff, on which are marked the lengths of the offsets of the several curves required to be set out; and likewise the lengths of the halves of such offsets. Then having settled the first portion of his line, which will doubtless be straight, the level-pegs may be considered as fixed points, through which he is, as nearly as possible, to run his curves; this therefore is an appropriate place to explain the mode of setting out curves.

To set out Railway Curves.

It is manifest that no Railway can be constructed throughout, in one straight line; it is also obvious that two straight lines for a Railway cannot be connected at an angle; it is therefore necessary to connect straight portions of railway lines by curves, as they are called. By a certain law of nature, if a body be projected in free space, by the force of gunpowder or any other impulse, it will by the force of this impulse, in conjunction with the law of gravity, describe the curve line of a parabola; the parabola would hence appear to be the most proper curve to have adopted for Railways; but it would have been inconvenient both to lay down on paper, and to set out on the ground. Segmental curves, that is, segments of circles, have therefore been adopted; but these when running into, or out of, straight lines, should be tangential to those lines; and when connected with curves of different radii, whether of similar or of contrary flexure, must be tangential to such other curves.

The perpendicular to the radius, at its extremity, is a tangent to the circle, by Prop. XVI. Book iii. Euclid's Elements.

If the circumferences of two circles touch each other internally, (that is, if the segments of circles of different radii, but similar flexure touch each other,) their centres and the point of contact are in the same line, by Prop. XI. B. iii.

And if the circumferences of two circles touch each other externally, (that is, if segmental curves of contrary flexure touch each other,) their centres and the point of contact are in the same line, by Prop. XII. B. iii.

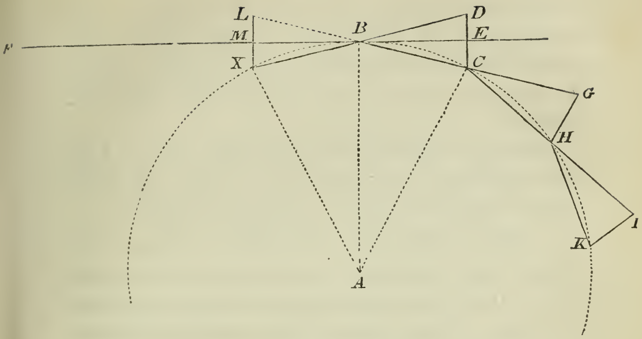
Hence to lay down on paper a tangent to a curve, at a given point, the radius of the circle being drawn to the required point, a line at right angles to the radius at that point will be the tangent required.

And to lay down a curve tangential to a given curve, at a given point; let a line be drawn through the centre of the circle and the given point in the periphery, and let the radius of the proposed curve be measured, from the point of the periphery, on this line.

It is obvious, from what has been above adduced, that this rule holds good whether the proposed curve be of greater or of less radius, than the given one, or of similar or contrary flexure.

But in the field, curves cannot be struck from centres, on account of the lengths of the radii; other means must therefore be had recourse to, for marking the peripheries.

The Institution of Civil Engineers of London have been pleased to signify their approval of the following system of setting-out Railway-curves, proposed by the author, so far as to award him a Telford Premium, for a paper submitted to them on the subject.



PROPOSITION I.

To set out a curve of given radius.

In the accompanying figure let A be the centre of the circle, a segment of which is to be out; F E a tangent at any point B; B X, B C equal chords; let B D be taken equal to B X, and join D and C, by the line cutting the tangent in E.

Then the triangles B A X, B A C, are equal and similar by construction; also the triangles D B C, B A C, are similar; for B E being a tangent to the circle, and B X, B C, drawn cutting the circle from the point of contact, the angles X B F, C B E will equal the angles in the alternate segments of the circle (Euclid, B. iii, Prop. 32); that is, D B E and C B E equal respectively $\frac{1}{2}$ X A B, and $\frac{1}{2}$ B A C, the angles at the centre.

Therefore in the similar triangles B A C, D B C

$$AB : BC :: BC : DC;$$

that is, radius : chord :: chord : offset,

$$\text{or, offset} = \frac{(\text{chord})^2}{\text{radius}}$$

Hence if the common four-pole chain be used to set out a curve, the length of the offset at the extremity of every one chain chord produced will be known, and the point in the periphery marked, from the preceding simple construction.

Suppose the radius of the curve is 100 chains, and the chord one chain; then reducing all to inches (the statute four-pole chain containing 792) we have, as above—

$$100 \times 792 : 792 :: 792 : \text{offset } (= 7{\cdot}92).$$

If any other standard of measure than the four-pole chain be used the radius and chord must be expressed accordingly. Hence whatever chain or standard of measure is used we have the following rule:—

Divide the number of inches in that chain by the number of such chains in the radius of the curve, and the quotient will be the offset required in inches.

PRACTICE.

Let it be required to set out a curve of 100 chains radius.

Stake out any tangent line, as F B; then the distance of the point C from this line produced to E (one chain), for a one-chain chord of the curve, will be $\frac{1}{2}$ D C, or 3·46 inches; now one end of the chain being held at B, the position of the other end C, will be ascertained by measuring 3·46 inches by means of an offset-staff, from the point E. That is, first measure one chain's length from B to E, and mark the point E, by a pole, or otherwise, and having one end of the chain held at B, move the other end from E, in the required direction, viz., towards C; the distance E C being at the same time measured by an offset-staff, duly divided for the purpose.

As the angle B E C is a right angle, an inconsiderable error is incurred by setting out an isosceles, instead of a right-angled triangle; but this would not be remedied in practice by setting out a right angle, because B E is not truly a full chain in length.

The first point being determined, measure one chain on the chord B C produced; that is, determine the point G, and if the end C is held fast, whilst the end G is moved to H, through the length of the full offset due to the required curve, measured with the staff as before, the point H is determined; and if C H be produced to I, the point K may be determined in the same manner; and so on, by this simple use of the chain and offset-staff.

The offsets for curves of radii varying from 5 to 320 chains are given in page 13 of this volume.

PROPOSITION II.

To pass from a straight line into a curve of given radius.

This proposition is included in the preceding, supposing F B to be the line from which it is required to pass into the curve B C H; the intermediate offset to the point C, and the successive points H K, &c. may be determined as already shewn.

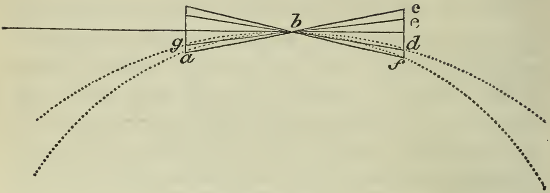
PROPOSITION III.

To pass from a curve into a straight line.

Let K C B, (same fig.) be the curve from which it is required to pass into the straight line B F. Produce C B to L, that is, measure one chain forward; and as the distance L M equals C E, let that distance, viz. half the offset of the given curve, be measured in the required direction, as before; when the line B M, being tangential to the curve, and in the desired direction, may be produced at pleasure.

PROPOSITION IV.

To pass from a curve of given radius to one of greater or less radius.



Case 1.

From a curve of given to one of less radius, as from the curve *g b d* to the tangential curve *a b f*, of less radius.

Proceeding as before, *the offset will be the greater offset minus half the difference of the offsets.* Thus, if the chord *g b* is produced to *e*, the offset required will be *e f;* but *c f* equals the greater offset, *e d* the less, and *c e* is half their difference.

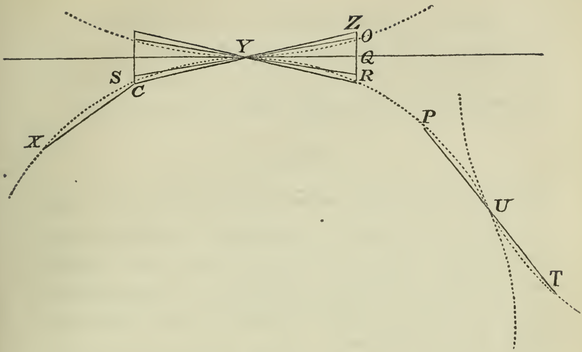
Case 2.

From a curve of given radius, to one of greater radius, as from the curve *a b f* (same fig.), to the tangential curve, *g b d*, of greater radius.

The offset at the point of change will be, *the less offset plus half the difference of the offsets.* Thus, if the chord *a b* is produced to *c*, the required offset will be *c d*, which equals *d e* + *e c;* but *d e* equals the less offset, and *e c* half the difference of the offsets.

PROPOSITION V.

To pass from a curve of given radius and flexure into one of contrary flexure.



Case 1.

From a curve of given radius to one of greater.

In passing out of the curve X Y into the tangential curve Y O, if the chord C Y be produced to Z, so that Y Z equals C Y, then the distance to be set off will be Z O; but as has been already shewn, Z Q equals half the offset due to the curve X Y, and O Q half that of the curve Y O; hence it is evident, that at the extremity of the first chain of the new curve it is necessary to set out half the difference, (or perhaps more obviously, *the difference of the halves) of the respective offsets due to the two curves.*

When passing out of one curve into another of greater radius, as in the present case, this half difference must be set out, *in the same direction as if proceeding with the first curve*, as from Z to O.

Case 2.

From a curve of given radius to one of less radius.

When passing out of the curve O Y (same fig.) into the curve Y X; if the chord O Y be produced to S, so that

Y S equals O Y, the distance to be set off will be S C, which again is half the difference of the offsets; but in this case, it is evident the half difference must be set out on *the same side as the succeeding offsets of the new curve.*

Case 3.

From a curve of given radius into one of the same radius.

In this case, as there is no difference of offsets, so there is no distance to be set off, and *the two chords constitute one right line*, as P U T (same fig.). Of course this line must not be taken at either a greater or a less length than double the chord in use.

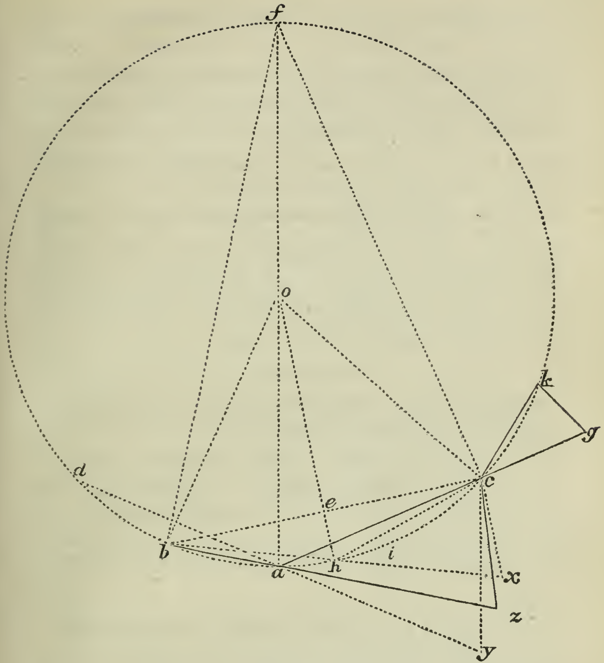
If the turn at Y or at U, be too short for the ground, a common tangent should be interposed between the curves.

Proposition VI.

To set out a curve when an obstruction occurs at a given point, as at i (fig. 88).

In this case, as it is inconvenient at the point i to set out the usual chord, let any other chord be taken, as $a\,c$. Then $b\,a$ being the one-chain chord, let it be produced to z, and draw $a\,c = a\,z$; from the point a, draw the diameter $a\,f$; draw $c\,f$ and $b\,f$, and the chord $b\,c$; bisect the arc $b\,c$ in h, and draw the radii $o\,b$, $o\,h$, and $o\,c$; draw $b\,h$, and $h\,c$; make $h\,x$ equal to $h\,c$; draw $a\,d$ equal to $a\,c$; connect the point c with x, z, and y, and those three lines, viz., $c\,x$, $c\,z$, and $c\,y$, will be in continued proportion; so that having found the first and third of them, the second being a mean proportional, will be known also.

Now $b\,f\,a$ and $a\,f\,c$ being right-angled triangles, $b\,f = \sqrt{a\,f^2 - a\,b^2}$, and $c\,f = \sqrt{a\,f^2 - a\,c^2}$; and $a\,b\,f\,c$



being a quadrilateral inscribed in a circle, the rectangle under the diagonals is equal to the sum of the rectangles under its opposite sides, [Proposition C X, Cooley's Geometrical Propositions]; or $af \times bc = ab \times cf + ac \times bf$; therefore $bc = \frac{ab \times cf + ac \times bf.}{af}$

In the right-angled triangle oeb, ob equals the radius, be equals half bc, and $oe = \sqrt{ob^2 - be^2}$; hence oe, and consequently eh, are known; and thus cx, which equals twice eh, is found.

The angles $b\ a\ c$ and $b\ h\ c$, being in the same arc of the circle, are equal; therefore their complements, $c\ a\ z$ and $c\ h\ x$, are equal; but those complements are respectively the vertical angles of isosceles triangles, therefore those triangles are similar, consequently $c\ x$ is to $c\ z$ as $c\ h$ is to $c\ a$; now $c\ x = \frac{c\ h^2}{o\ h}$, and $c\ y = \frac{c\ a^2}{o\ a}$ [Proposition I.]; that is, $c\ x : c\ y :: \frac{c\ h^2}{\text{rad.}} : \frac{c\ a^2}{\text{rad.}}$, consequently as $c\ h^2$ is to $c\ a^2$; but $c\ h^2 : c\ a^2 :: c\ x^2 : c\ z^2$; therefore $c\ x : c\ y :: c\ x^2 : c\ z^2$; then, multiplying the means and the extremes, $c\ x \times c\ z^2 = c\ y \times c\ x^2$, and dividing both sides by $c\ x$ we have $c\ z^2 = c\ y \times c\ x$, and therefore $c\ x : c\ z : c\ y$.

PRACTICE.

Suppose $b\ a$ one four-pole chain, and $a\ z$ two chains in length; and the radius 100 chains; then

$$b\ f = \sqrt{158400^2 - 792^2} = 158398{\cdot}02, \text{ and}$$

$$c\ f = \sqrt{158400^2 - 1584^2} \quad 158392{\cdot}08, \text{ and}$$

$$a\ f \times b\ c = a\ b \times c\ f + a\ c \times b\ f = 376{,}348{,}991{\cdot}04; \text{ and}$$

$$b\ c = \frac{376{,}348{,}991{\cdot}04}{158400\ (=a\ f)} = 2{,}375{\cdot}94; \text{ and } \frac{b\ c}{2} = b\ e = 1187{\cdot}97.$$

Then

$$o\ e = \sqrt{o\ b^2 - b\ e^2} = \sqrt{6{,}272{,}640{,}000 - 1{,}411{,}272{\cdot}7209}$$

$$= \sqrt{6{,}271{,}228{,}727{\cdot}280} = 79{,}191{\cdot}089; \text{ and}$$

$$e\ h = o\ h - o\ e = 79{,}200 - 79{,}191{\cdot}153 = 8{\cdot}911 = \frac{c\ x}{2};$$

therefore $c\ x = 17{\cdot}822$, and $c\ x^2 = 317{\cdot}624$; and $c\ y$ (by Proposition I.) $= 31{\cdot}68$.

But $c\ x : c\ y :: c\ x^2 : c\ z^2$, therefore $17{\cdot}822 : 31{\cdot}68 :: 317{\cdot}624 : 564{\cdot}600$; and the square root of $564{\cdot}600$ is 24, which is the value of $c\ z$, the offset required.

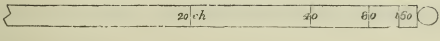
It will be found by a similar computation, that the offset for the extremity of the next following one-chain chord, as at g, should be 12 inches, viz. half the length just ascertained.

If the chord, instead of being produced two chains' length, be taken half a chain in length, then the offset will be 2·97 inches; and for the next following full chain double as much, or 5·94 inches.

It will be perceived that *the offset for double the chord, is rather more than three times the offset for the one-chain chord;* and *that for the half chord, rather more than one-third of the usual offset.* Hence it appears that, practically speaking, a good approximation only can be arrived at, if the usual length of chord be departed from. For permanent work, therefore, all obstructions should be removed, previously to setting-out the curves of a line of railway.

The Surveyor then being acquainted with the foregoing rules, and provided with a copy of the Table of Offsets,* will find it advisable to stick one of his ranging poles in the ground, and having a clean piece of wood as an offset staff, held with one end against such pole, to measure from the centre of the pole, and mark on the staff the offsets of such curves as he is likely to require, and also the halves of such offsets.

It may be observed that the offset of a given curve is equal to half the offset of a curve of half the radius; thus the offset for a curve of 80 chains radius, is equal to half the offset for one of 40 chains. It will be found convenient, and less liable to error, to divide the staff, not into feet and inches, but into the respective offsets themselves, and to distinguish the several marks by their own particular Radii; as in the annexed fig.



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* Page 13.

Then, as there ought not to be any other marks on the staff, with common care errors may be avoided. By dividing a staff in this manner it will not be necessary, nor advisable, to denominate any offset half of another, but to mark every one as a full offset, and where a half is required, to set out by the mark denoting that of double the required radius.

Great care should be taken to keep the chain true, and always to stretch it fairly; likewise to place the poles truly, and to keep them perfectly upright;* as a very trifling error in either of these respects, will at certain points, lead to serious deviations from the true curve. If a pole takes a cast it should be at once thrown aside. As soon as a curve is settled, good pegs should be driven into the several points marked by the poles.

A line having been set out with due care in the manner here described, and a section of it taken, all necessary deviations may readily be made; and, that having been done, the line may be considered permanently determined.

WORKING PLAN.

The curves may then be laid down on the plan, together with the base line; when, it is obvious, the survey may readily be completed with a chain only, without any other instrument, by means of the Base and the Permanent line. It will not be necessary to extend it beyond two or three chains at each side of the line, when that is sufficient to define the several enclosures.

The location of each peg should then be defined on the general plan; and the section being carefully taken over those pegs, will afford the means of preparing the "Property Plans;" and also the contract plans and sections.

This plan should be laid down at two chains to an inch.

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* A watch suspended by a guard-chain makes a convenient plump-bob.

Working Section.

The working section should be plotted to the same horizontal scale as the plan, and 20 feet to an inch for the vertical heights. On this section a line must be drawn to represent the surface of the earthwork below the ballast. This is sometimes called the "line of forming," or the "ballast line." Indeed the "rail line" had better be altogether omitted in this section, because it is of no real use, and not unfrequently leads to confusion and error.

The earthwork and widths must all be calculated from the lower line, or line of forming.

Property Plans.

These, as the title denotes, are plans of the several properties, through which the line passes; an appropriate plan of his property being prepared for each proprietor. Each plan should, if possible, contain the whole of one owner's property. If the working plan has been laid down to a scale of two chains to an inch, the property plans may either be transferred from that, or in general more conveniently, reduced to a scale of four chains to an inch.

The quantities should be computed from the "widths," in figures, and not from the plan itself.

Each plan should have an appropriate title, stating the name of the railway, of the particular parish, proprietor, tenant, &c. The "reference" will contain a list of the several fields, distinguished by their respective numbers, on the Parliamentary and the general plans; together with the quantity in each, &c.

Widths.

In a level district the widths may be ascertained by computation, thus,

From the field-book, make a list of heights at each peg, which will generally be sufficient; but if the height between any two pegs differs materially from that at either of those pegs, or from the mean, then the greater height or depth should be taken. A given height being multiplied by double the slope, and having the width of the base or centre, together with the width reserved for fences at each side, all in feet, added to the product, the sum will be the width required at that peg. Then if the several widths in a given enclosure be added together, and a mean taken, that mean reduced to links and multiplied by the number of chains, will give the content.

The heights being in feet, the widths will be in feet also. It is true that if the vertical scale is 20 feet to an inch, and the heights be measured by a scale of 30, they will be in links, very nearly; but it is usual, and less liable to error, to measure by the same scale as was used in the plotting, and to reduce the whole width to links afterwards, for convenience in computation and in plotting. A fraction of a chain may be computed from the plan.

When the ground at one side of a line is higher than at the other, it will be necessary to take cross-sections, for the purpose of computing the widths of land and the quantities of earth-work; and as it is not unusual to lay one line of rails first, before proceeding with the other, the centre of the line intended to be first executed, will probably be that set out on the ground; these circumstances must be attended to in laying down and setting out the widths. From one or both of these causes, the widths at both sides are therefore seldom the same; a list in the following form will be found convenient.

WIDTHS OF LAND.

No. of chain.	Heights.	East side in Feet.	West side in Feet.	East side in Links.	West side in Links.	Total in Links.	
0	5	28.5	38.5	44	59	103	Embankment.
1	8	33.	43.	50	65	115	
2	10	36.	46.	55	70	125	
3	15	43.5	53.5	66	81	147	
4	9	34.5	44.5	52	67	119	
5	3	25.5	35.5	39	54	93	
6	0	21.	31.5	32	47	79	
7	4	27.	37.0	41	56	97	Cutting.
8	8	33.	43.0	50	65	115	
9	2	24.	34.0	36	51	87	
10	1	22.5	32.5	35	50	85	

REMARKS.

N.B. The line on the plan represents the centre of the Eastern line. There are 10 feet added on the West side for the other line, and 11 feet at each side for fences. The whole width of the base or centre is 30 feet. Slopes 1½ to 1.

A surveyor having prepared his paper as in the foregoing example, and carefully taken the heights from his field-book,* will find the calculation of widths a very simple matter. It is the best way to take one column at a time; thus, as above, multiply the height, 5 feet, by the ratio of slope, viz. 1.5 to 1, the product 7.5, is to be added to the constant quantity 21 feet, making 28.5, as set down. The constant quantity, 21, is composed of 10 feet of the base or centre, and 11 feet for fences. A portion of that column being done, it is evident that the adjacent one may be filled in, by merely adding 10 feet to each width, as in the example; for if 10 feet of the base be appropriated to the east side, it is obvious that 20 remain for the west side; and the fencing is the same on both sides. The feet may then be reduced to links by adding to each quantity one half of itself, as already mentioned. Here also if a portion of the eastern column be done, the western one may be filled up by adding 15 to each eastern width. The quantities of land will be calculated from the column of "total widths."

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* It is better to have too much land than too little, therefore in setting down heights when a fraction of a foot occurs, the next higher foot should be set down, and never the next lower.

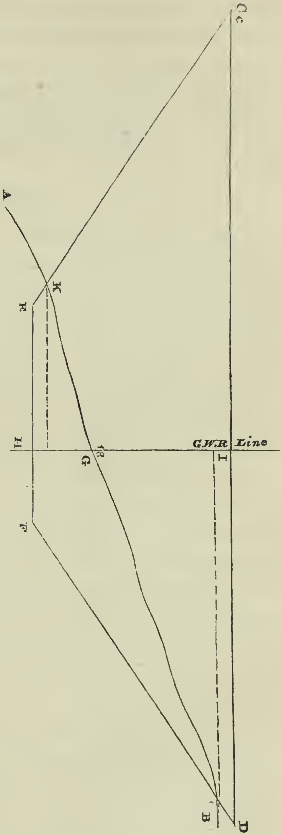
Each peg being marked on the plan, the respective widths may readily be laid down from the list. The section will afford data for locating the pegs.

Then the surveyor being provided with a tracing of the plan, and an abstract from the list of widths, will be enabled to set them out on the ground. He should not trust to his eye for setting them out at right angles to the line, but should use either an optical square, or the chain as already directed.

TRANSVERSE OR CROSS-SECTIONS.

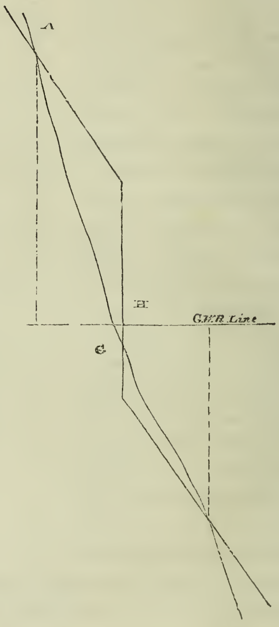
Cross-sections are required whenever the ground is higher at one side of a line than at the other. They should all be taken in the same direction, that is, suppose, as we have before supposed, a line from London to Bristol, then the cross-sections should be so taken as to face in the same direction as the longitudinal section; that is, towards Bristol. This would be accomplished by starting at the London end, and commencing each cross-section at the south, or left hand side, of the line, and plotting from left to right. Each chain-peg being similarly numbered on the plan and on the longitudinal section, the same pegs should be distinguished by the same figures in the cross-sections; thus, if in the plan, a peg be numbered 65, this same number 65 should distinguish it in both the longitudinal and cross-sections. It frequently happens that one cross-section is applicable for several chains; this should be distinctly noted in the field book.

The requisite cross-sections having been taken, should all be laid down in one direction as described, to a large scale. A very convenient scale is 20 feet, for both horizontals and verticals.



The simplest practical mode of ascertaining the widths from cross-sections, is to draw the cross-section of the railway on the cross-section of the ground, plotted to a large and equal scale as above. And the most expeditious mode of accomplishing this is to cut a piece of drawing paper, by the same scale, into the form of the railway transverse section, as C D E F, with a vertical line drawn through it, to represent the trig. line, or line of pegs; then the cross-section of the ground's surface being plotted with a similar line, to represent the same point, and the height or depth, of embankment or cutting, at the given peg, being marked on such vertical line of the surface section, the railway section may be applied to it, by making the extremity of the vertical line in the one coincide with the point marked on the other. For example, suppose the line A B, in the figure, to represent the cross section of the ground's surface at the chain peg No. 18, where the cutting is 8 feet, then from the peg, at G, mark on the vertical line, as at H, the 8 feet cutting, to which point bring the extremity of the vertical line of the railway cross-section; when the required figure may readily be drawn in, and the respective widths at each side be measured, from the vertical line to the points of intersection, as B and K, of the cross-sections.

To the quantities so found, the quantity reserved for fences must of course be added.



If the line had been in embankment it would have been necessary merely to invert the railway cross-section, and apply it to the given point on the vertical line as before.

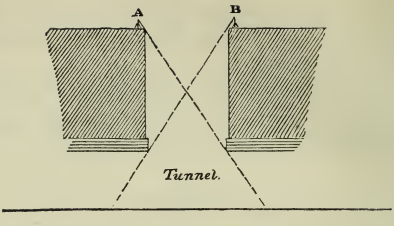
If the line were on the level, or nearly so, of the surface, it would be necessary to place the railway cross-section first for the cutting, and then to reverse it for the embankment, as shewn in the annexed figure. In this case the vertical line should be drawn at both sides of the paper.

If the widths only were required, it would not be necessary to draw in the railway section, as it would be sufficient to mark the points of intersection; but when it has been drawn in, the same diagram may be used for calculating the earthwork.

For calculating earthwork generally, see Mr. Macneill's tables for that purpose; but they will give an approximation only, where cross-sections are necessary.

TUNNELS.

The setting out of a tunnel requires much care. It is a matter of no small difficulty to keep the centre line true, the work being generally carried on from both ends and from intermediate shafts at the same time. It is necessary in the first place to set out the line very accurately on the surface. It should be produced so as to be defined by good marks beyond both extremities; these marks should be outside the work altogether, so as not to be liable to disturbance, from slips or otherwise, before the completion of the tunnel. When the line is straight it will require only ordinary care to keep it true at the ends; and if it is not very deep, the usual way is, having sunk a shaft, to fix two plumb lines in the centre line at the surface, in the mouth of each shaft, and to have them long enough to range from below. This mode was adopted on most of the railways in this kingdom; but in the Box tunnel, on the Great Western Railway, the shafts were so deep, (some of them from 300 to 400 feet,) that it was found the plumb lines would not answer the purpose; on account of oscillations caused by currents of air or otherwise; the following method was therefore resorted to, viz. shafts 20 feet in diameter, were sunk, in the line; the centre line at these shafts was fixed by a theodolite, or a transit instrument, as the case might be; the mode of accomplishing this will be understood on reference to the figure. The instrument being first

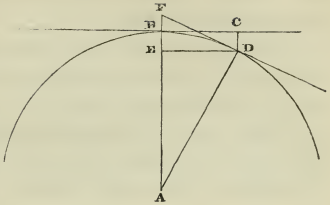


set in the line, on the surface, at A or B, and a point fixed in the bottom of the tunnel by means of the vertical arc; and then another point found in a similar manner from the other side; a short length of line was thus obtained, which was carefully produced both ways, to meet other portions worked from the adjacent shafts. These points, and the line through them, were tested at every length, before the brick-work was put in. When a shaft is so deep that the range of the arc of a theodolite is insufficient to enable an observer to see to the bottom of a tunnel, a transit instrument must be made use of instead.

On the London and Birmingham, and other lines, high towers were built, to enable an observer to see from one end to the other of a tunnel; this appears to be a heavy expense to incur for a temporary purpose.

To set out a Curve in a Tunnel.

The system already given for setting out curves is not practicable in tunnels for want of room, but the following may be adopted:—a shaft being sunk at the point of junction of the tangent and the curve, and the tangent or straight line being marked in the bottom, as also the point of junction, let the straight line be produced for a given distance, and at the extremity of the line so produced, let the offset for the given curve be set out at right angles.



The required offset may be found thus :—in fig. 92, let B be the commencement of a curve, and B C the straight line produced; it is evident that C D will be the offset required. To find the length of it, draw D E parallel to B C; D E will then be perpendicular to B A; and B E equal to C D; and, E being a right angle, A D^2—E D^2 (= B C) = A E^2, thus we have A E; and then A B (radius) — A E = offset required.

EXAMPLE.

Required the offset for a curve of 10 chains radius, the tangent produced being one chain.

Inches in 10 chains.	Inches in one chain
$7920^2 = 62726400$	$792^2 = 627264$
627264	
	7920
62099136	$\sqrt{62099136} = 7880.3$ ft. in.

offset required . . . . . . 39.7 = 3 . 3 $\frac{7}{10}$

It will be perceived that the offset thus found, equals half the offset given in the table for a ten chain curve, agreeably to proposition I, this Section.

Having set out the first offset, as C D, and also marked E B equal and parallel to it, produce E B to F, that is, make B F = B E, and a line drawn from F through D, will evidently be tangential to the curve at D, and may be proceeded with as before; and so on as far as may be required.* The distance of one chain has been taken for the example, although that is much more than can be accomplished in practice.

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* It is scarcely necessary to remark, that curves may be set out on the surface, on this principle, and sometimes with convenience; as, when two

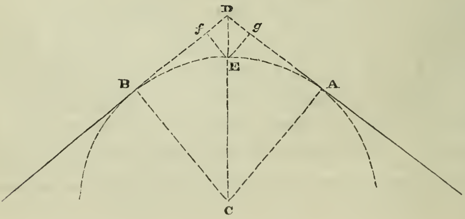
A heading, or small tunnel, is frequently driven through the site of a tunnel to ensure accuracy in the setting out; but it is a tedious, expensive, and unsatisfactory operation; and unless required for drainage, had better be omitted. It is not at all unusual to have the tunnel through, before the heading can be made available.

GRADIENTS.

To set out the gradients of a line, let a picket be driven to the required level at any given point; then the rise or fall per chain being ascertained from the table of gradients, at page 11 of this volume, may be set out by means of a spirit level and staff.

straight lines are to be connected by a curve. In that case it will be well to produce the straight lines, until they meet, as at D, in the annexed figure; then having found the angle A D B, and assumed a radius, as A C, for the curve, the length of the line D A, or in other words the point of contact A, may readily be found by trigonometry; because having found the angle A D B, we have of course got its half, viz. A D C; then D A C being a right angle, we have got the three angles, and one side, A C, of the triangle A C D, to find the side A D. As C D bisects the vertical angle, that line being defined by pickets on the ground, will afford a good means of checking the work; for if the distances from A to *g*, and from B to *f*, be divided into equal parts, say chain lengths, and the respective ordinates be set off as above directed, the offsets *f* E and *g* E should meet in the line D C, as at E.

It is manifest, that if the end of either of the tangents be fixed, by reason of any obstruction, the length of the appropriate radius, may be found in a similar manner; for we shall still have the three angles, and one side of the triangle, A C D.



CANALS.

To the selection of a line for a canal, the same general principles apply, as for a railway; but in addition thereto, care must be taken to obtain, if possible, sufficient water at the summit-level, to allow two locks-full of water to every boat likely to pass in every 24 hours; that is, a lock-full at each end of the pond; that so a boat may be passed from one extremity of the canal, to the other, without avoidable delay.

There should be sufficient water to allow for leakage and waste also. These may be taken at two locks-full per day at each end.

Exhalation seldom exceeds ten inches, between rains.

PARLIAMENTARY SURVEY.

The same clauses of the "Standing Orders," apply to canal surveys as to railway surveys, unless where the term "Railway," only is used; and in addition it is directed,

"That in all cases where it is proposed to divert into any intended cut, canal, reservoir, aqueduct, or navigation, or into any intended variation, extension, or enlargement thereof, respectively, any water from any existing cut, canal, reservoir, aqueduct, or navigation, whether directly or derivatively, and whether under any agreement with the proprietors thereof, or otherwise, the notices shall contain the name of every such existing cut, canal, reservoir, aqueduct, or navigation, the waters supplying which by virtue of any Act of Parliament, will either directly or derivatively, flow or proceed into such intended cut, canal, reservoir, aqueduct, or navigation, or into any intended variation, extension, or enlargement thereof."

"That in all cases where it is proposed to make, vary, extend, or enlarge any cut, canal, reservoir, aqueduct, or navigation, the plan shall describe the brooks and streams to be directly diverted into such intended cut, &c. or any variation, extension, or enlargement thereof, respectively, for supplying the same with water; it shall also exhibit the height of the several embankments, and the depth of the several cuttings respectively, on a scale specified thereon." (see fig. 1. pl. III.)

"And in cases of bills for improving the navigation of any river, there shall be a section which shall specify the levels of both banks of such river, and where any alteration is intended to be made therein shall describe the same by feet and inches."

To be enabled to exhibit the height of the several embankments, &c. of a canal, &c. it will evidently be necessary to take cross-sections; because the heights at both sides will frequently differ very materially; indeed it is difficult to discover the wisdom of this order; the surface of one bank of a navigable river or a canal is generally pretty well defined by the towing-path, but the opposite bank may frequently be called anything, as in many cases scarcely any two men would measure to the same distance from the water, for the bank; nor does the object to be gained, if the matter were ascertained, very distinctly appear. Those who wish to comply with the spirit of the order will denote one bank by a full line, and the other by a dotted one; but both banks may be shewn by similar lines.

Permanent Line.

The instructions given for a railway permanent line, are with the obviously necessary modifications, applicable here.

Docks and Reservoirs.

Docks and reservoirs are subject to the same standing orders as canals; under the head of canals, therefore, the reader will find the requisite information.

"That the plan may exhibit the height of the several embankments, and the depth of the several cuttings respectively," it will be necessary to take transverse sections right across the work at every material change of height of the surface. The sectio-planography may be drawn in from such cross-sections. The mode of proceeding will be evident on reference to Plate IV.

By the wording of the standing orders it is not necessary to show the cross-sections, otherwise than in sectio-planography.

SECTION VI.

TRIGONOMETRICAL SURVEY.

General outline of the system.

AN accurate survey, for any extensive operation, such as the measurement of an arc of a meridian, or for the formation of a chorographical map, showing the positions of towns and villages, the boundaries of counties or other divisions, together with the roads, rivers, and other important features: —or a plan for military purposes,—must necessarily be based on an extended system of triangulation, the preliminary step in which is the careful measurement of a base line. At each extremity of the base, angles are taken to several surrounding objects previously fixed upon as trigonometrical stations; and also, when practicable, the angles subtended at each of these points by the base itself. The distances of stations, from the ends of the base, and from each other, are then calculated, and laid down on paper, forming so many fresh bases, from whence other points are trigonometrically determined; until the whole tract of country to be surveyed is included in a net-work of triangles, of as large a size as is proportioned to its general extent, and to the quality and power of the instruments at command. Within this principal triangulation a set of secondary triangles is formed, the sides found by calculation, and laid down in like manner. A third series, of still smaller triangles may be formed, proceeding from the most convenient and shorter lines of the secondary set. The whole, or nearly the whole, of the sides of this last set; for an accurate survey, must be measured with the chain, and by their means the interior detail is to a great extent filled up; the remainder, as roads and rivers, being obtained by traversing with a theodolite and chain; or by sketching, with the assistance of some portable instrument. The degree of accuracy required, will determine which of these methods is to be adopted; the former is practised on the Ordnance Survey of Ireland, but the latter was considered sufficiently minute for that of England.

In a flat uncleared country, any attempt at a system of triangulation would be useless. In such a case the only mode of ensuring tolerable accuracy, in surveys of great extent, is that which has been generally adopted in the construction of geographical maps. The latitudes and longitudes of a number of the principal and most conspicuous stations are determined by astronomical observations, and the distances between them calculated, that so their relative positions may be laid down as correctly as this mode of fixing them admits of. In surveying an extensive line of coast, where the interior has not been triangulated, no other method presents itself; and a knowledge of practical astronomy, therefore, becomes indispensable. A topographical survey further requires that some of the party employed upon it should be well versed in the general outlines of geology; as a correct description of the soil and mineral resources of the different parts of every country forms one of its most important features. The heights of the principal hills, and of marked points along the ridges, plains, valleys, and water-courses, above the level of the sea, should also be determined; on the Ordnance Survey of Ireland this is done by levelling with the theodolite; that is to say, by the angles of elevation and depression. In a survey of less pretensions to correctness in minute detail, the heights may be ascertained barometrically.* A sketch of a certain tract of country, on a far larger scale than that of most general maps, is constantly required in war-time, for the purpose of showing the military features of the ground, the relative positions of towns and villages, and the nature and direction of the roads and rivers comprised within its limits. This species of sketch, termed a "military reconnaissance," approaches in accuracy to a regular survey, in proportion to the time and labour that is bestowed upon it. Those who desire to pursue this branch of the subject are referred to Lieutenant Frome's work,* in which they will find all the necessary information, and to which useful work this article is much indebted.

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* Vide p. 119 et sequitur.

The system of forming the "net-work of triangles," already alluded to, of as large a size as is consistent with the circumstances under which the survey is undertaken, is to be considered as the working out of a general principle, to be borne in mind in all topographical and geodesical operations; the spirit of which is, always to work from whole to part, and never from part to whole. By the former method errors are subdivided, and time and labour economised; by the latter, the errors inseparable from even the most careful observations are constantly accumulating, and the work drags on at a slower rate and an increasing expenditure.

Measurement of a Base Line.

In fixing upon an appropriate site for the measurement of a base line for a trigonometrical survey, a level plain should obviously be selected, where both ends of the base would be visible from the nearest stations. Steel chains, glass,† deal, and platinum rods, have at different times been used for the purpose of determining their lengths;

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* Outline of the method of conducting a Trigonometrical Survey, for the formation of Topographical plans, &c. Weale, 1840.

† Vide Appendix.

but either of these units of measurement, whichever is preferred, must be supported in such manner on trestles that it may lie perfectly straight.* The line thus forms a portion of a great circle, which has ultimately to be reduced to its proper measure at the level of the sea, and at one mean temperature.

The exact measurement of a base, although apparently easy, is perhaps the most difficult, and the most important part of a trigonometrical survey, as upon its accuracy that of every triangle depends. As the mode adopted by Colonel Colby to obtain the accurate value of the base measured on the Ordnance Survey of Ireland,† by the beautiful contrivance resorted to for counteracting the effects of expansion and contraction, between the points marked at the extremities of the metal rods, which were used for the purpose, instead of allowing for the alteration produced in their length by every change of temperature, is a new adaptation; as the mode of obtaining a visual, instead of an actual contact‡ is the most correct; and as the method itself is perhaps susceptible of a greater degree of accuracy than any that has been practised, either in England or on the Continent, a short description of it will not be unacceptable.

Two bars, one of iron and the other of brass, ten feet long, placed parallel to each other, were rivetted together at their centres; see Plate V; it having been previously ascertained, by numerous experiments, that those metals expand and contract, in their transitions from cold to heat, and the reverse, in the proportion of three to five. The brass bar was coated with a non-conducting substance, to equalise the susceptibility of the two metals to change of temperature; and across each extremity of these combined bars was fixed a tongue of metal, with a minute dot of platinum, almost invisible to the naked eye, and so situated on these tongues, that, under every degree of expansion or contraction of the rods, the dots at each end always remained at the constant distance of ten feet apart. This will be better understood by reference to fig. 1, Plate V. A is the iron bar (about five-eighths of an inch wide, and an inch and a half deep), the expansion of which is represented by three; B the brass bar, of the same size, the expansion of which equals five; the two being riveted together at the centre C. D E and *d e* are the tongues, pinned on so as to allow the bars to expand; the platina dots are near the extremities D and *d*. The tongues are, by construction, made perpendicular to the rods, with the dots ten feet apart, at a mean temperature of 60° Fahrenheit; and the expansion or contraction taking place from the common centre C, when A expands any quantity which may be expressed by three, B expands, at the same time, a quantity equal to five, and the position of the tongues is changed to D F and *d f*, the dots D and *d* remaining unalterably fixed at their original distance apart.* It is evident, from the construction, that these dots could not, if desired, be brought into either contact or coincidence; but a more correct plan was adopted; which consisted of laying each bar so that the dot at its extremity, should always come within a fixed distance of the end of the next bar. This was effected by means of a small apparatus consisting of two microscopes, shewn at *a a* fig. 3, Plate V; each furnished with cross hairs, attached to a similar compound bar, *b*, six inches long, mounted on a stand, by means of which they could be laid perfectly horizontal, by a spirit level, *c*; the cross hairs of the microscopes attached to these bars occupying the position of the dots on the longer ones.

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* See Appendix.

† For an account of the operations on the Survey of England, see Appendix.

‡ This mode was first applied by Sir George Shuckburgh to the comparison of English standard measures. Vide Philosophical Transactions for 1798.

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* In most methods of measuring a base line the temperature requires to be noted as every chain or rod is laid, (see Appendix) the mean of all the readings of the thermometers is ultimately taken; the temperature in which the chain or rod was originally proved being known, the difference between it and this mean is readily found; and the expansion or contraction for one degree of heat being also known, that quantity is multiplied by the number of degrees constituting the difference, and the resulting quantity is added or deducted, as the case may be. For example, in the measurement of one base on the English survey, the sum of all the degrees shewn by the thermometers, placed by the side of the 100 feet chain, was 98511; wherefore $\overline{\frac{98511^\circ}{5} - 54^\circ \times 272.8} \times \frac{0.0075}{12} = 3.1069$ feet. There having been five thermometers for each chain's length; the chain having been originally

The point of starting was a stone pillar, with a platina dot let into its centre, with a transit instrument placed over it, by means of which the line was laid out with the greatest precision, with the assistance of sights at each end of the coffers. The first coffer having been carefully levelled, by means of the levels shown at L L, figs. 2 and 3, and the microscope-stand so adjusted as to have the cross hairs of one microscope over the starting point, the adjacent point of the first bar was brought under the cross-hairs of the other microscope,* by means of three slow-motion screws, attached to the coffer, in which each bar was fixed; so that the coffer could be moved on either side, backwards or forwards, as required. They were laid on trestles, T T, fig. 3, equidistant from their centres, that they might always have the same bearing.

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measured in 54° of temperature; the length of the base being 272.8 chains; and 0.0075 of an inch being the expansion for 100 feet of blistered steel to 1° of Fahrenheit.

* The microscopes commonly used stood six inches apart; but to meet cases where the uneven surface of the ground, rendered it difficult to bring the bars to a level at this distance, the interval was occasionally altered. Microscopes of different lengths were used when the form of the ground rendered it necessary to lay the coffers on different levels, so that the platina dots might be brought in the focus of each microscope.

A stand with screws, was similarly adjusted at the other end of the first bar, the second coffer laid and levelled, and its adjacent dot brought under its microscope; the third, fourth, and fifth bars, similarly adjusted, and thus 52 feet were measured. The first coffer was then carried forward, and similarly placed in front of the fifth bar, and the operation repeated, and so on *ad finitem.*

An average distance of about 250 feet was measured per day; five coffers, or a length of 52 feet, being levelled and laid together. About 400 feet of this base were measured across the river Roe, clumps of pickets having been driven, by a small pile engine, at intervals of about 5 feet 3 inches apart from centre to centre, on the heads of which the coffers were rested. At the end of each day's work, a triangular stone was sunk at the end of the last bar laid, with a cast iron block fitting into it, having a brass plate, adjustable by screws, with a dot let into it. This dot was brought exactly under the cross-hairs of the extreme microscope, and served as a starting point the following day; a sentinel being always left in charge of the stone, which was secured by a wooden cover screwed over it. The total length measured of this base amounted to about eight miles; two miles were subsequently added by a method to be presently described, making the entire distance between the two extremities, rather more than ten miles.

An account of various modes of conducting the measurement of base lines, may be found in Airy's "Figure of the Earth," in the Encyclopædia Metropolitana, and also in "Puissant, Traité de Geodesie," and other French works; as well as in the Appendix to this volume.

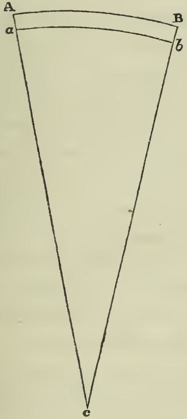
A base measured on an elevated plain, is thus reduced to its proper measure at the level of the sea. Say, as the mean semi-diameter of the earth, augmented by the height of the measured base above the sea, is to the mean semi-diameter, so is the measured base, to the reduced base, at the level of the sea. Or,

Call A B the measured base at a given elevation A *a*, above the level of the sea - - B
a b, its value at the level of the sea - - *b*
C *b*, the mean radius of the earth - - R
And A *a*, the altitude above the level of the sea *h*

Then $R+h : R :: B : b$; therefore $b = \frac{R\,B}{R+h}$,

And the difference of the measured and reduced base,

$$B - b = B - \frac{B\,R}{R+h} = \frac{B\,h}{R+h}. = \frac{Bh}{R}$$ very nearly



The radius of the earth may be considered equal to 21,008,000 feet; then, if to the *log.* of the base, in feet, be added the *log.* of the altitude; and the *log.* of the sum of radius and altitude be subtracted therefrom, the remainder will be the *log.* of a number to be deducted from the measured base, to reduce it to its value at the level of the sea. This correction, though generally of small amount,* is not to be neglected when the base is measured on ground of any considerable elevation.

Other methods are given for making this correction, but the above is deemed sufficient.

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* See Appendix.

If the surface slopes uniformly, the mean height may be taken;* if it is very irregular, it may be divided into several parts.

In measuring a base for the topographical survey of any small portion of land, it will be sufficient, for ordinary purposes, to measure its length carefully, two or three times, with a good and correct chain; and to take an accurate section of the line; from which it may be reduced, by trigonometry, to its true horizontal value. The line should be first set out with a theodolite, from one extremity to the other. A base, the length of which is to be subsequently determined by means of glass rods, compensation bars, or otherwise, is, in general, first measured in this manner.

During the progress of the measurement of a base, a few points in it should be carefully determined, and marked, by driving strong pickets, or sinking stones, into the ground, with the exact termination of the chain clearly and accurately defined upon them. These marks serve for testing the different portions, and comparing them reciprocally with each other. The length of the base on the Ordnance Survey of Ireland, was not attained entirely by measurement, an addition of two miles being made to its measured length, by calculation, as already stated. This calculation was also contrived to answer the purpose of verifying the measurement of intermediate portions of the base, between marks left for the purpose, in the following manner, viz.

In the accompanying figure, let A B represent the portion of the base actually measured, and B C that to be added by calculation, for the purpose of extending the base to C, to obtain a more eligible terminus.

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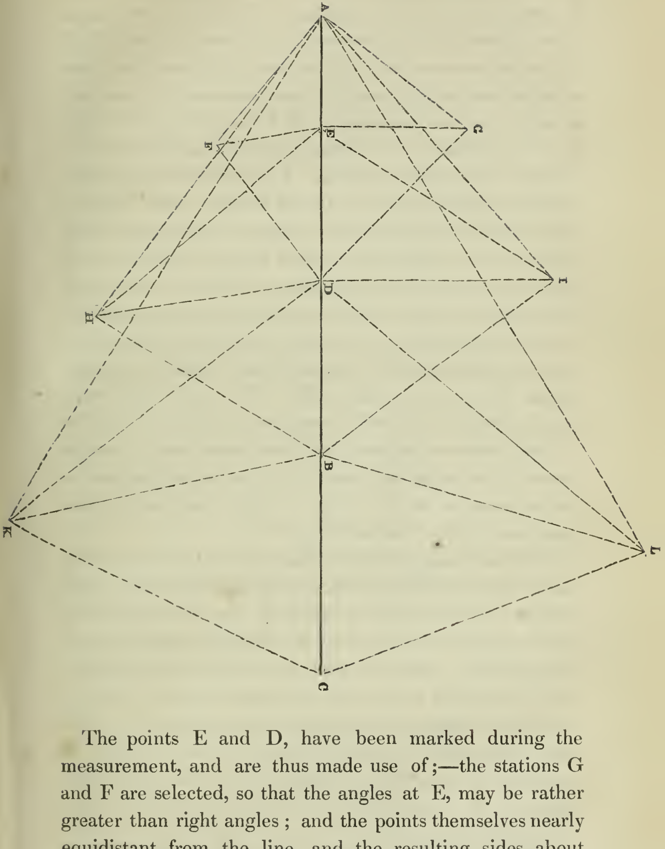
* See Appendix.



The points E and D, have been marked during the measurement, and are thus made use of ;—the stations G and F are selected, so that the angles at E, may be rather greater than right angles ; and the points themselves nearly equidistant from the line, and the resulting sides about equal to A E. Similar conditions determine the positions of H, I, K, and L. At A the whole of the objects, visible, are most accurately observed with a large theodolite, which is then taken to the other points on the line, as well as to those selected on either side of it; by means of which all the angles are measured. Then from A E, and the three observed angles, G E and E F are determined; from each of which, in the triangles, G E D and F E D, the side E D is obtained. The distances thus found form two checks on its measured length. I D and D H are in like manner, calculated from A D, and likewise from E D, as bases; and each of these again furnishes data for the checking of D B. Lastly B L and B K are found, from A B, and also from D B; from the mean results of which, B C, the required addition to the measured base, is obtained.

The whole length of the base having been determined, and the trigonometrical stations fixed upon, the triangulation is to be proceeded with. The most conspicuous points should be selected as stations, due regard being had to their relative positions; as the nearer these triangles approach to being equilateral, the less will be the error in the calculation of the sides, resulting from any minute inaccuracy in the observed angles.

The base being generally of inconsiderable length, compared with the distances between the points of the principal triangles to be ultimately deduced from it, the sides of these triangles must be, from the first, gradually increased, as rapidly as is consistent with the remark in the last paragraph, until they arrive at their greatest limit;* which, in an extensive survey, is determined by the distance at which these points can be rendered distinctly visible. In 1822, the reflection of the sun from a plane mirror was employed in Hanover, for the purpose of rendering distant stations visible; and a similar method was adopted by Colonel Colby and Captain Kater, in verifying General Roy's triangulation for connecting the observatories of Greenwich and Paris. The station on Hanger Hill tower could not be seen from Shooters Hill, although only 10 miles distant, owing to the dense smoke of London; but was rendered clearly visible by tin plates attached to the signal post, so as to reflect the sun towards the station at stated times on a certain day. The same plan was tried the following year at the station on Leith Hill, near Dorking, and succeeded in rendering the station visible at the distance of 45 miles, although the hill itself was never once seen. The utility of thus employing the sun's reflected rays being established by these results, an instrument was invented by Lieut. Drummond, Royal Engineers, in lieu of the former temporary expedients, for directing the rays upon the station to be illuminated, a description of which will be found in his paper on the means of facilitating the observations on distant stations, published in the "Philosophical Transactions" for 1826. But a contrivance was still wanting to produce a light sufficiently brilliant to answer for distant stations at night. Bengal lights had been used by General Roy; which were succeeded by Argand lamps and parabolic reflectors; and these again, by a large plano-convex lens, prepared by Messrs. Fresnel and Arago, and used by the latter gentleman in conjunction with Colonel Colby and Captain Kater, and by means of which a station, distant 48 miles, was observed. The light invented by Lieut. Drummond, and described in the above mentioned paper, far surpasses all previous contrivances. A ball of lime, about a quarter of an inch in diameter, placed in the focus of a parabolic reflector, and raised to an intense heat by a stream of oxygen gas directed through a flame of alcohol, produced a light eighty times as intense as that given by an Argand burner. A station on a hill in the barony of Ennishowen, of great importance, could not be seen from Devis mountain, near Belfast; consequently this instrument was sent there by Colonel Colby; and, in spite of boisterous and hazy weather, the light was brilliantly visible at a distance of sixty-seven miles, and evidently would have been so at a much greater distance.

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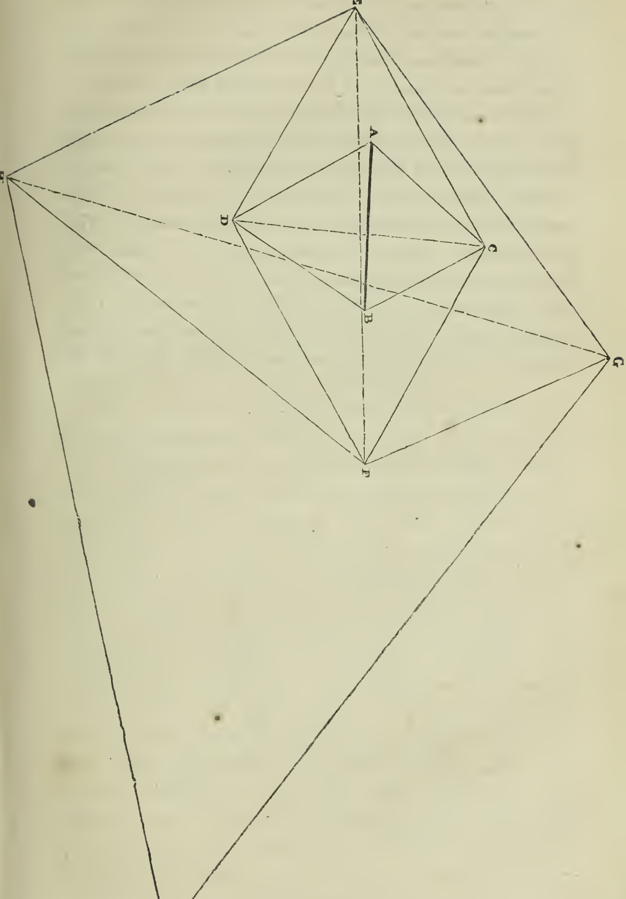
* The lengths of the sides of some of the triangles on the Ordnance Survey of Ireland exceed 100 miles, all deduced from the original base of about ten miles.

It has been already stated, that the sides of the principal triangles should increase as rapidly as possible, from the measured base. The accompanying sketch will show how this may be managed without admitting any ill-conditioned triangles.

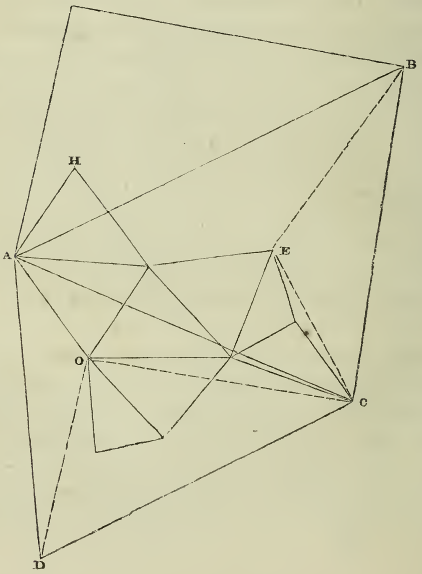
A B is supposed to be the measured base, of two or three miles, or as long as can conveniently be obtained; and C and D the nearest trigonometrical points. All the angles being observed, the distances of C and D from the extremities of the base are very carefully calculated. Then in each of the triangles, D A C and D B C, we have two sides and the contained angles, to find D C; one calculation acting as a check upon the other. This line, D C, is again made the base from which the distances of the stations E and F are computed from D and C; and the length of E F is afterwards obtained in the two triangles E D F and E C F. In like manner the relative positions of the points H, G, I, &c. are obtained; and some such system should be pursued until the stations arrive at the required distance apart.

On the Ordnance Survey, both of England and Ireland, a large instrument, three feet in diameter, made by Ramsden, was used for fixing the principal stations. The angles of the secondary triangles were observed with a twelve-inch theodolite, mounted on a repeating stand; which is an instrument susceptible of being used with minute accuracy. The sides of these triangles were, on an average, about ten


E
A
H
D
C
B
G
F
I

or twelve miles in length. These triangles were again divided into smaller ones, with sides of from one to three miles long; a smaller theodolite, of seven, and in some cases even of five inches diameter, being used for measuring their angles.

All the stations of the secondary series which were fixed upon during the progress of the principal triangulation, were observed with the largest instrument; and a number of minor points, as mills, churches, &c. were observed with the twelve inch theodolite, from different stations. Thus the connection between the three classes was established; and many of the minor stations, the positions of which had been determined by calculation from the secondary triangulation, were checked by being made the vertices of minor triangles. For example, the point E in fig. 95, had



been determined from the base B C; and O from both D C and A D. Thus forming a connection between the secondary and minor orders of triangles, and so constituting a check upon the latter.

The length of the sides of the smallest triangles must depend upon the intended method of filling in the interior. If the contents of parishes, estates, &c. are to be computed, the distances between the points must be diminished to one or two miles, for an enclosed country; and to two or three perhaps, for one more open. If no contents are required, and the object of the triangulation is solely to ensure the accuracy of a topographical survey, the distances may be augmented according to the degree of minuteness required; the scale upon which the work is to be laid down being governed by the same consideration.

One or more meridian lines, according to the extent of the work, should be accurately ascertained and laid down.*

On the continent the instrument that has been generally used for measuring the angles of the principal and secondary triangles is Borda's repeating circle, which is described in French works; but the theodolite is universally preferred in England; and those of the larger description, in their present improved state, are, in fact, portable altitude and azimuth instruments. The theodolite possesses the great advantage of reducing instrumentally, the angles taken between objects situated in a plane oblique to the horizon, to their horizontal values; but this reduction with any instrument measuring the exact angle between two objects having different altitudes is a matter of calculation. Dr. Pearson has given a formula for this correction, which formula, together with a demonstration of the problem, are given by Lieut. Frome, but it is not considered necessary to repeat them here.

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* See Appendix.

SPHERICAL EXCESS.

All observed horizontal angles are essentially spherical angles; and in every triangle measured on the surface of the earth, the sum of the three angles, must, if taken correctly, be more than 180°. The lines containing the observed angles are in fact *tangents* to the sphere (supposing the earth to be one); whereas, to obtain the three points considered as vertices of a plane triangle, the angles must be reduced to the value of those contained between the *chords* of the arcs constituting the sides of the spherical triangle. The correction for this spherical excess, though too minute to be applied to angles observed with moderate sized instruments, being completely lost sight of in the unavoidably greater errors of observation, should nevertheless be calculated in the principal triangles; which is easily done on the supposition that the area of a spherical triangle, whose sides are immeasurably small, compared with the whole sphere, may be considered identical with that of a plane triangle, whose sides are of the same length as those of the spherical one; and whose angles are each diminished by one third of the spherical excess. From this theorem, demonstrated by M. Legendre, and known by his name, is deduced the form $\frac{S}{R}$; or for the excess in seconds $\frac{S}{R^2} R''$: where S denotes the area, and R the radius of the earth.*

The earth being considered a perfect sphere whose radius is 21,008,000 feet; one degree on the earth's surface being 365,154.6 feet, one second = 101.43 feet, and 101.43^2 = the square feet in a square second. R the radius = 206,264.8* seconds; and the expression becomes $\frac{\text{area of triangle in feet}}{(101.43)^2 \times (206{,}264.8)^2} \times 206{,}264.8$; or in logarithms, *log.* area of the triangle — 4.0123486 — 5.3144251 = *log.* area in feet — 9.3267737 = *log.* excess in seconds; that is to say, from the log. of the area of the triangle taken as a plane one, in feet, subtract the constant log. 9.3267737, and the remainder is the log. of the spherical excess above 180°, in seconds, nearly.

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* R'' may be considered identical with $\frac{1}{\sin. 1''}$. See "Puissant Traité de Geodesie." Vol. I. p. 100.

On the Trigonometrical Survey of England the spherical excess was constantly calculated, not solely for the purpose of diminishing the observed angles by the amount, but to correct the observations. Thus in one of the large triangles in Dorsetshire, the sum of the three angles was 0″.5 less than 180°; the calculated spherical excess amounted to 1″.29; shewing an error of 1″.79 in the observation; and in many of the triangles this error was more considerable. When the sum of the three observed angles is less than 180°, one-third of the error thus found, added to each of the angles, corrects them as angles of a spherical triangle; and one-third of the spherical excess deducted from each of these corrected spherical angles converts them into the angles of a plane triangle, ready for calculation, the sum of whose angles is 180°. The following details of the above mentioned case will render this more evident.

Observed Angles.		One third of Error.	Corrected Spher. Angles.	One third of Sph. excess.	Rectilinear Ang. corrected for calculation.
Maker Heights	45° 54′ 37″	+ .597	45 54 37.597	— .43	45 54 37.167
Bolt head	48 39 24.5	+ .597	48 39 25.097	— .43	48 39 24.667
Butterton	85 25 58	+ .597	85 25 58.597	— .43	85 25 58.167
	179 59 59.5		180 0 1. 29		180. 0 0

When the sum of the three observed angles exceeds 180°,

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[*] These are the figures made use of on the Survey of England.

the amount of the excess should be added to the spherical excess, and one-third of the whole deducted from each of the observed angles. By this means, it will be perceived, the error of observation, as well as the spherical excess, is distributed over the three angles.

The spherical excess might be calculated for each angle separately, by reducing the angles of the spherical triangle, to the angles formed by the chords. (Woodhouse, page 239.)

Thus there are three modes of solving the large triangles of a survey ; first, by calculating them as spherical triangles, with the corrected spherical angles ; secondly, by computing them as rectilinear triangles with the angles of the chords ; and thirdly, by Legendre's method of reducing each angle by one-third of the spherical excess ; this last method is by far the most expeditious. In the "Base du Systeme Metrique,"* the sides of the triangles were computed by all three methods. On the Ordnance Survey they were mostly calculated by the second, and checked by the third.

This subject is treated at length in Woodhouse's Spherical Trigonometry, and in various other works.

When the theodolite cannot be placed exactly over the station, a correction for this excentricity, termed the "Reduction to the Centre" becomes necessary. Lieut. Frome gives a pretty mode of performing this.

In observing the angles for triangulation, too much care cannot be bestowed upon the adjustments of the instrument. The peculiarities in the construction and management of the larger instruments, with which such angles are generally observed, are soon understood by any person conversant with the adjustments of the smaller class of instruments in general use; the adjustments of which have been already given.

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* A French work detailing the operations of Messrs. Mechain and Delambre, for determining not only the figure of the earth, but also some certain standard of measure. These commissioners devoted much time and labour with a view to establish a standard of weight as well as of measure ; and a law was passed founded on their deductions. But they having adopted $\frac{1}{334}$th as the earth's compression, which is now known to be inaccurate, their metre scientifically speaking is not correct ; and old established customs having been found too strong for the new law, their labours have not resulted in the practical advantages anticipated.

Large instruments are usually furnished with four, and sometimes with six verniers, the mean of all of them should invariably be taken, and each angle repeated six or eight times. The errors of excentricity, and graduation of the instrument, are thus almost annihilated, and those of observation of course much diminished. The repetition of angles is the only means by which they can be measured with any degree of accuracy with small instruments.

It is frequently necessary to refer to trigonometrical stations long after the angles have been observed, either for the purpose of fixing intermediate points, or of rectifying errors that may have crept into the work. Large, marked stones, or good oak pickets well driven, should therefore be always buried under the principal stations, which are not otherwise identified by permanent erections; and a clear description of the relative positions of these marks, with reference to objects in their vicinity should always be recorded. If a good oak peg or picket, about twelve inches or more in length, be driven to within an inch or two of the surface, it may be found within any reasonable time.

A method of finding a lost station is given by Lieut. Frome,* to whose work the reader requiring it is referred; but if a station be properly marked and described at first, it may be found for years afterwards.

The triangulation should never be permanently laid down on paper until its accuracy has been tested by the actual measurement of one, or more, of the distant sides of the triangles, as a base of verification; and by the calculation of others from different triangles, to prove the identity of the results.*

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* Page 24.

But it will be found very satisfactorily to assist in laying out the ground, to get a rough diagram laid down as soon as possible, from the best data that can be obtained; indeed this should not be by any means neglected, as it will save much time, and very probably some oversights.

The triangles should always be laid down by taking the lengths of the sides in beam compasses, and never by protraction of the angles.

When the skeleton triangulation is completed, the next step is the interior filling in of the country, the delineation of roads, &c. either entirely or partially, by measurement. This will be understood by studying the figure, plate VI. A B C and B C D, are supposed to be two triangles of the minor series; the interior boundaries are fixed by the chain lines *a a*, *b b*, &c. and the roads by the lines, *c c*, *d d*, &c.; which have been worked by a theodolite and chain as already described. It will be perceived, that both the boundary, and the road surveys, are connected with the triangulation, and consequently with each other, thus forming a connected whole.

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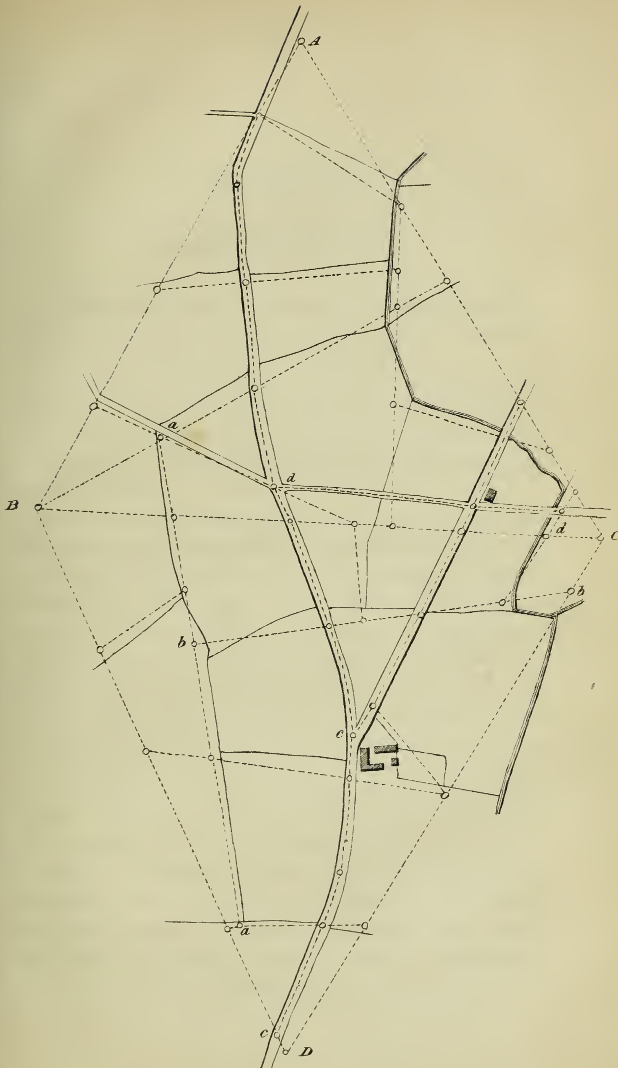
* See Appendix, "Trig. Survey of England."



Madeley lith. 3 Wellington St. Strand

SECTION VII.

Subterraneous Surveying.

Underground surveying, as already mentioned,* has been hitherto almost universally performed with the magnetic needle, that is to say, with the Circumferentor.

It appears so monstrous, that such a vast amount of human life, and of valuable property, should, at this time of day, be left dependent on so variable an instrument, that the statement would be almost incredible, if the fact were not so notorious.

It is to be hoped that at no very distant period the Theodolite may be substituted for the Circumferentor. It is not to be expected, that men, who so seldom use their instruments on the surface of the earth, will exercise the care and caution necessary to ensure even an approximation to accuracy with the Circumferentor. Doubtless there are exceptions, but as a general rule it may safely be asserted, that mining surveyors do not duly test their needles. Even Mr. Fenwick, "colliery viewer and surveyor of mines," in his elaborate "Treatise on subterraneous surveying, and the magnetic variation of the needle," has given the variation at London only, and not at Newcastle, where his work was published ; from which it may fairly be inferred, that the amount of the variation at Newcastle was not known. A Circumferentor needle ought to be tested for variation, at least once every year ; but it may be questioned whether or not the majority of mining surveyors test their needles, for that purpose, once in a life-time.

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* Vide p. 99.

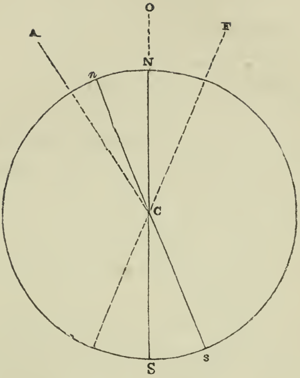
As long as the Circumferentor is continued in use the utmost care should be taken to keep it in good order; and as the ascertaining of a true meridian requires time and trouble, the bearing of any other permanent line may be taken in lieu of it, and laid down on the plan. Thus let the bearing of the line connecting two distant pit-mouths, be carefully taken, with the needle in use, and laid down on the working plan, instead of a nominal true meridian. If the bearing of such a line were so taken regularly, once in every year, the annual variation might be well ascertained, and so one source of error avoided. But as soon as any difference of variation was observed, that is, as soon as it was observed that the line fixed upon as a meridian, or datum, or test line, did not bear exactly as it did when the plan was plotted, all the bearings should be reduced to that line, as to a true meridian, by the following rule.

TO REDUCE BEARINGS FROM A MAGNETIC, TO A TRUE MERIDIAN.

Let N S, in the accompanying figure, represent the true meridian, N being the North; and *n s* a magnetic needle, playing freely on a centre, C. The needle being perfectly at liberty, and coming to rest in the line *n s*, that line will represent the magnetic meridian; and the arc, *n* N, the angle of variation; that is, if the angle N C *n*, be 22° 30′, then the needle will have 22° 30′ variation; and as *n* is on the left-hand, or West side of N, the variation is Westerly.

Suppose the same circle to represent a Circumferentor, and that the bearing of the object O, with the true meridian is required. If *n s* be the needle representing the magnetic meridian, and the object O bear "North 22

30′ East," the variation, as already assumed being 22°, 30′ West, then deducting the amount of the variation, from the observed angle, the bearing of O with the true meridian will be due north.



If the bearing of an object at A, with the true meridian were required; and its bearing by the needle were North 10° West; then as the variation is West, the observed bearing must be added to the amount of variation. Hence 10°. + 22° 30′ = 32°. 30′, is the angle that C A makes with N C S.

The reader must observe that the true bearing of any object is nothing more than the angle the object makes with the true meridian, instead of that which it forms with the magnetic meridian.

Therefore when an object bears to the West of the magnetic North, the variation being westerly, the amount of the variation is to be added to the observed bearing, as above shewn; because the object stands more distant from the true meridian than from the magnetic, by the amount of the variation.

When the observed bearing is to the East of the magnetic North, and exceeds the amount of the variation, then the variation is to be deducted from the observed bearing. Thus if the bearing of the object F, were N. 45°. E., and the variation as before; 45°.—22° 30′ = 22° 30′; the angle to be plotted from the true meridian, would be N. 22° 30′ E, instead of 45°. as observed.

If the amount of the variation exceeded the observed bearing, and were on the contrary side, then the bearing should be deducted from the variation. Thus suppose the observed bearing to be N. 10°, East; then 22° 30′ — 10°. = 12° 30′; and N. 12$_{o}$ 30′ W, would be the required angle from the true meridian. So that in this case, although the observed angle reads E. of N, yet it must be plotted to the West of the true meridian.

It is evident that magnetic bearings might be as readily reduced to bearings from any other line as from a true meridian. Indeed if a line were assumed, more nearly in the magnetic meridian, it might save an apparent anomaly; as in the case of a bearing being a little to the east of north. However, let a line be selected, which is well marked by permanent objects; as by two pit-mouths as already directed; or by the main pit-mouth and the parish church spire; and let a large stone be sunk in this line produced to a considerable distance, say ten or twenty chains, the farther the better; let a mark be cut in the stone, and the Circumferentor be set up over that mark, and the bearing of the line carefully taken, laid down, and distinctly described on the working plan, as a meridian line. Then, if some such precaution has not been already taken, a complete new survey of the whole of the main lines of levels should be made, and the magnetic bearing of the new invariable meridian, or test line, carefully laid down, with the amount of its variation, or difference from the magnetic meridian, recorded in figures on a new working plan. When additions to the plan are required, the magnetic bearings of the fixed meridian line should be observed with the needle about to be used; and if such bearing does not coincide with the bearing recorded on the plan, the bearings of the lines to be added should be reduced to bearings from the fixed meridian, and protracted accordingly.

If these precautions were observed, the annual variation of the magnetic meridian would not vitiate the plan, however great the length of time during which the work might be in progress. The surveyor would nevertheless still have to contend with diurnal variation, local attraction, and the inherent imperfections of the instrument.

When a telescope is attached to a Circumferentor, it should be used throughout the survey, or not at all; because the collimation of the telescope seldom coincides with the collimation of the instrument. That is, the axis of the telescope, seldom coincides exactly with the centre line of the compass box.

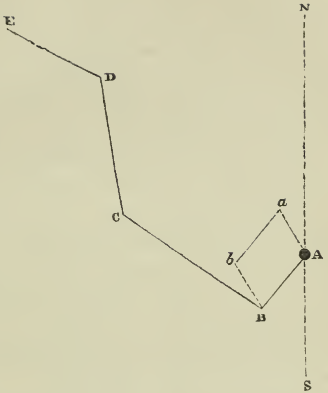
The principles on which a subterraneous survey is performed, are the same as those on which a surface survey is made, with an instrument for taking the angles: the student may therefore be referred back to page 85 for the theodolite; or to page 93 for the circumferentor.

To commence, however, it is necessary that a person go forward, in the direction of a line to be surveyed, with a lighted candle in his hand, to the utmost distance his light can be seen through the sights of the instrument; its bearing is taken, and noted in the field-book; the distance is then to be measured, the chain leader holding a lighted candle and his end of the chain in the same hand; when, by keeping the two lights in conjunction, the chain follower may measure the line truly. The follower should go forward before the leader proceeds, otherwise, the arrow or mark may not easily be found; the follower should not carry his end of the chain in his hand, but let it be drawn after him.

A survey of the surface, over the workings, should be made simultaneously with the underground survey, and connected with it; that so the position and progress of the workings, with reference to the boundary and other surface lines, may be known.

When the workings are carried on by means of levels, the subterraneous and surface surveys may readily be very well connected; for, let a point be taken at the entrance, or mouth of a level, and a line be assumed, which shall pass through such point, both ways, that is, into the level and out of it; let the outer end be produced as far as may be necessary to enable the surveyor to see over the top of the level, as well as into it; by which means, it is evident, lines may be run on the surface, corresponding with those underneath it, great care being observed to measure both series of lines as nearly horizontal as possible. As long as the Circumferentor continues to be used, both surveys should be made with the same instrument, and, as nearly as convenient, at the same time.

When works are carried on by means of pits or shafts, the connection of the over-ground, with the under-ground survey cannot be effected so mechanically as by levels. Instead of adopting the same mode of commencement as for entering at the mouth of the level, make any place, as *a*, on the surface, near the pit A, the point of commencement, and run off from that point, *a*, the first bearing and distance of the subterranean survey, in the same manner as if *a* was the centre of the pit A; which first bearing and distance say is, South 30° West, 4 chains; and is represented by *a b*. Before the instrument is removed from *a*, take the bearing and distance of the centre of the pit A, which suppose to be South 30°. East, 3 chains; *a* to A. Now remove the instrument to *b*, and from thence turn the sight in the direction of South 30° East, and run off 3 chains, as from *b* to B; then the line A B being parallel to *a b*, and



running from the centre of the shaft, A, represents truly on the surface, the course of the subterraneous excavation. The instrument being then removed to B, the second line may be run off, in its true position, by the bearing and distance, as underground; and so of the others, from C and D.

The surveyor must be careful to keep the underground workings regularly surveyed and plotted, as they proceed; say at every one, two, or three months. By thus having the entire of the workings laid down, the most eligible directions may be pointed out, by the needle, for proceeding with, or stopping, the workings; lines may be run so as to meet, or to intersect each other, &c.; but it is evident, that the success of these operations depends mainly on the correctness of the instrument, and on the care with which it is used. Numerous instances, however, are on record in which excavations have missed the end proposed; producing irretrievable disgrace, vexation, and expense. But the missing of a desired point, is not so much to be wondered at, as that it should ever be arrived at, by means of the Circumferentor.

The working plan should be laid down to a large scale, say two chains to an inch; and if it should become necessary to continue the plotting on a separate sheet, from that on which it was commenced, marginal lines should be drawn either parallel, or at right angles, to the meridian, on both sheets; when, these lines being kept accurately in coincidence, the work may be correctly carried forward. No surveyor should be content to produce a station line, and lay down his meridians from that. If it be required that the sheets be permanently connected together, it should be done before the work is carried across the joint, and not afterwards, by any means, as it is all but impossible to connect them afterwards without distorting the lines. The sheets should be connected with varnish if possible. If varnish is not readily obtainable, glue should be used, laying on as much as is necessary, and no more. Paste should not be employed on any account, as it is so subject to deterioration by the atmosphere.

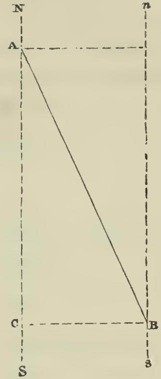
The Reduction of a number of Bearings and Distances to one Bearing and Distance.

The mining surveyor will frequently find it advisable in the plotting of subterraneous surveys on the surface, for the purpose of fixing a point, to reduce the bearings and distances to one bearing and one distance. To effect this, it will be necessary for him to understand the following

DEFINITIONS.

I. Meridians are north and south lines, supposed to pass through every station, parallel to each other.

II. The difference of latitude, or the Northing or Southing of any station line, is the distance that one end of the line is north or south from the other end; or it is the distance which is intercepted on the meridian, between the beginning of a station line and a perpendicular drawn from the other end to that meridian. Thus, if N S in the annexed figure, be a meridian line passing through the point A of the line A B, then is A C the difference of latitude, or southing of that line.



III. The departure of any station line, is the nearest distance from one end of the line, to a meridian passing through the other end. Thus, B C is the departure, or Easting of the line A B.

But if C B were a meridian, and a station line were measured from B to A, then would B C be the difference of latitude, or Northing, and A C the departure of Easting of the line B A.

IV. That meridian which passes through the first station, is sometimes called the first meridian; and sometimes the first meridian is an assumed line, passing on the east or west side of the plan.

Example.

To find the difference of Latitude, and Departure, the Bearing and Distance being given.

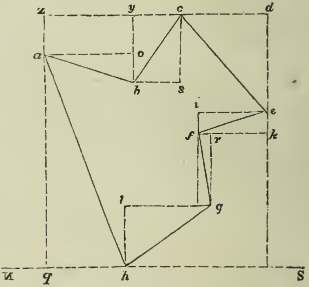
Let the bearing and distance of the line *a b*, in the next fig., be, S. 16° 45′ W., and its length 10 chains 12 links. Required *a o*, the difference of latitude, and *o b*, the departure.

By right angled plane Trigonometry.

R.	*a b*		S. *o a b.*	*b o*
90° :	10.12	: :	16°.45 :	2.919
10.0000000 :	1.0051805	: :	9.4596884	
	9.4596884			
	.4648689	=	2.9165 = *b o*, the Departure	

R.	*a b*		S. *a b o*	*a o*
90° :	10.12	: :	73°.15 :	9.696
10.0000000	1.0051805		9.9811711	
	9.9811711			
	.9863516	=	9.69606 = Difference of Lat.	

Thus the difference of latitude, or southing, of the line *a b*, is found to be 9 chains 69 links; and the departure, or westing, 2 ch. 92 links. In like manner the difference of latitude and the departure, may be found for each station distance, having the bearing and length thereof given.



In the above example, N S being a meridian line, all lines parallel thereto, which pass through the several stations, will be meridians also, as *a o*, *b s*, *c d*, &c. And the lines *b o*, *c s*, *d e*, &c. perpendicular to those, will be east, or west lines, or departures.

In a complete traverse, or close, the northings will equal the southings, if the work be correct. Thus $ei + gl + hq = ao + bs + cd + fr$.

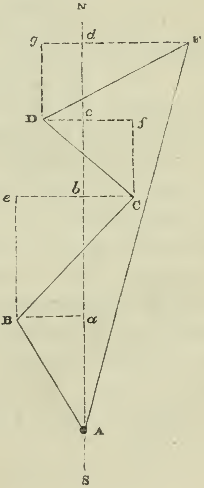
For let the figure be completed, and it is evident that $gl + hq + rk = ao + bs + cd$; and $ei - rk = fr$. If to the first part of this equation be added $ei - rk$, and to the latter part fr, then $gl + hq + ei = ao + bs + cd + fr$; that is, the sum of the northings is equal to the sum of the southings.

And the eastings equal the westings, viz. $cs + qa = ob + de + if + rg + lh$.

For $aq + yo\,(az) = de + if + rg + lh$; and $bo = cs - yo$. If to the former part of this first equation be added $cs - yo$, and to the latter bo, then $cs + aq = ob + de + if + rg + lh$; that is, the sum of the eastings is equal to the sum of the westings. Q. E. D.

By this means the accuracy of a traverse may be tested. But if a complete circuit shall not have been made, and it be required to determine the bearing and distance of one point, from another, as the point of F, from the pit A, in the annexed figure.

Let the northing and westing of A B; the northing and easting of B C; the northing and westing of C D; and the northing and easting of D F, be determined in the manner already exemplified; then if, as in this instance, the sum of the eastings exceeds the sum of the westings, deduct the less sum from the greater; and if there are southings as



well as northings, deduct the less from the greater of their sums also, and these two differences, that is, the excess of the northings over the southings, or *vice versa ;* and of the eastings over the westings, or *vice versa ;* will be the legs of a right-angled triangle ; the hypothenuse of which will be the distance required; its bearing and length may be thus found, viz.

Example.

	Dist.	Northing.	Easting.	Westing.
A B, North 30°. West .	5.50	4.76		2.75
B C, North 45°. East .	7.00	4.95	4.95	
C D, North 50°. West .	5.00	3.21		3.83
D F, North 65°. East .	7.00	2.96	6.34	
Sums .		15.88	11.29	6.58
Deduct westing .			6.58	

Difference, or easting of F from
the meridian of A = 4.71 = *d f*

Then the first northing *i. e.* 4.76 = A *a ;* the second, 4.95 = *a b*, &c. ; and the sum of the northings, 15.88, equals A *d*. The first westing, *i. e.* 2.75 = *a* B ; the first easting, viz. 4.95 = *e* C ; the second westing, 3.83 = *f* D ; the last easting, 6.34 = *g* F ; and the excess of the eastings over the westings, 4.71 = *d f.*

In this case, then, we have the sum of the northings, for one leg of a right-angled triangle ; and the excess of the eastings over the westings for the other leg ; from which data the required bearing and distance, of F from A, may readily be found, as it will be the hypothenuse of the right-angled triangle. First to find the bearing, or angle N A F, say

a d	:	Radius	::	*d f*	:	Tan. angle A
15.88	:	10.0000000	::	4.71	:	16°. 31′
3.2008505		2.6730209				2 6730209
		12.6730209				
		3 2008505				

9.4721704 = T.16°.31′ = Required bearing.

To find the distance say

R	:	*a d*	::	Sec. angle 16° 31′	:	A F
		3.2008505				10 0183005
		10.0183005				
		13.2191510				
		10.0000000				

3.2191510 = 16.56 = A F, the distance required.

Proceeding in this manner will, it is obvious, considerably diminish the chances of error; for in this case there would be only one bearing to be taken instead of four bearings; and one line to be measured instead of four lines. But if the mining surveyor is so wedded to old fashions as to prefer running lines on the surface corresponding with those below, or as it is called, "plotting his work on the surface," let him do so, but let him check it by the means here pointed out; and if he does not find the same point, by both modes, he may feel assured there is an error somewhere.

If the surface over the workings be very irregular, it will be advisable to select a level piece of ground, sufficiently large to contain the whole, and plot the work there; by means of which the bearing and distance of the required point may be found from any given point of the survey.

If proprietors choose to trust the management of their property to persons who are incompetent to apply these rules, they must, of course, abide by the consequences.

If the distances were accurately measured, the angles taken carefully with a good theodolite, and reduced to angles from the meridian,* and the whole reduced to one bearing and distance, a required point on the surface might be found with great precision. So that if it were required to sink a shaft from the surface down to any point of the workings, for drainage or otherwise, it might be accomplished by this means with credit and satisfaction.

"Traverse Tables," or Tables of "Difference of Latitude and Departure," or of "Difference of Latitude and half Departure," have been given by some authors, but they are necessarily very imperfect, because to be complete they should be very voluminous. I have therefore deemed it advisable to omit them altogether, and to recommend the surveyor to use Logarithmic tables.†

It is evident that all the points, B, C, D, and F, might be laid down from their respective differences of latitude and departure, if that were preferred to protracting the several station lines.

Mr. Fenwick states that "in making surveys where the distances measured are not horizontal, but declining, it will be necessary for the surveyor to reduce all his measurements to horizontal distances, which may be obtained by taking the angle that each separate distance makes with the horizon," &c. But he does not state by what means these vertical angles are to be obtained; nor is it very easy to conceive, whilst the surveyor's practice is confined to the circumferentor. With a theodolite the mode has been already explained in this volume.‡ An approximation may be obtained with a clinometer.§ But it is manifest that vertical angles cannot be satisfactorily obtained without a theodolite. Perhaps the best practical way for a mining surveyor will be, by means of a mason's level, carefully to

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* See p. 96. † See p. 44. ‡ See p. 145. § See p. 170.

ascertain the allowance due to a given distance, say one chain, in each inclination. Thus, let the chain be held fast at one end, on the inclined surface, and let a mason's level be held level at the same point, then by taking half the chain in one length, the elevation, above the surface, of the centre of the chain, may be found, by keeping it up to the same level as the end; a plummet let fall from such elevation will denote the point on the surface to which the half chain would extend horizontally; and of course the difference between that distance, and the half chain, measured along the surface, would be the quantity to be added for each half chain measured, throughout the inclination. But it will be the better way, to find the elevation due to the half chain, and to secure it, by a block of wood or otherwise, and then, taking the level forward, to hold it horizontal on such block, and so find the elevation due to the remaining half chain, and by letting fall a plummet as above directed, the allowance for each chain may be ascertained.* It is scarcely necessary to say that lines should be measured horizontally on the surface as well as underground. It is advisable to take the elevation of half a chain only, at one time, because the whole chain cannot be stretched straight, horizontally, in one length; but the surveyor must be careful to measure the two segments in the same right line.†

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* See p. 145.

† See p. 145.

SECTION VIII.

Marine Surveying.

It is not pretended in this volume to give a treatise on Marine Surveying, in the extended sense of the term, which embraces many details into which the surveyor, for whose use this treatise is intended, cannot be expected to enter. There are several treatises written expressly on this subject, to some of which, persons about to make a scientific enquiry will doubtless refer. That most recently published is by Mr. David Stevenson on "Marine Surveying and Hydrometry," applied to Civil Engineering, and is a most satisfactory work by a practical man.

To make a survey of an extensive line of coast, it will be necessary to triangulate it. To survey a very extended line of coast in a new country, the latitudes and longitudes of a number of leading points are generally determined by astronomical observations, as already mentioned.*

The general mode of effecting this has been already explained in Section VI., on Trigonometrical Surveying. The first thing to be done will be to select an appropriate site for a base line; which should be, or its extremities at all events, above high-water mark. If there be a flat strand a good position for a base will probably be found, without much difficulty; taking advantage of low water to measure such portions as may be covered when the tide is in. If a line cannot be had on the strand, a base must be measured on the most convenient and level piece of ground within the limits of the survey; a marsh in dry weather would answer the purpose admirably. Instances have occurred of measuring lines over the frozen surfaces of ponds or lakes, but in such cases care must be taken to complete the measurement, and to secure the terminations of the line before the breaking up of the frost.

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* Page 259.

In a survey of this kind it will seldom if ever be requisite to enter into these minute corrections which are necessary in a territorial trigonometrical survey ; it will generally be sufficient to measure the base line very carefully with a good steel chain made for the purpose. The chain should be accurately adjusted, and the line be measured at least three times ; the measurement should be made backward and forward, and the mean length of such measurements be adopted. The base should be taken of as great a length as possible, and must, of course, constitute one side of a triangle. A mode of extending the length of a base by calculation has been already explained, in Section VI., but its application, or not, will be governed by the extent of the survey.

Having measured a base line, the area of the intended survey should be gone over carefully, for the purpose of determining the trigonometrical points or stations. These should then be marked by long straight poles ; young larch, or Scotch fir trees are well adapted for this purpose ; they may be distinguished by flags, if necessary, but in general it will be found sufficient to whitewash them well, and attach a bunch of heath, or a small broom, to the upper end of each pole. The stations should be selected in conspicuous situations, that they may be distinctly seen from each other, and so situated that the sides of the several triangles may be as nearly equilateral as possible ; the average length of these sides will be determined by the extent of the survey, and the nature of the country. They should be taken of as great a length as practicable, that so the least time may be consumed in taking the angles, and as few chances of error afforded as possible. Stations should, if practicable, be fixed in all remarkable or important points on the coast about to be surveyed. By means of which, if its outline be reasonably regular, its survey may be accomplished. Where a minute survey of a particular portion is required, the line passing through or adjacent to it may be chained; and the details determined in a similar manner to that recommended for making a parliamentary survey.*

A rough diagram of the triangulation should be made as soon as the points have been resolved upon.

Then, all the stations having been duly marked on the ground, a theodolite should be taken to each in succession. The instrument being accurately adjusted over a station, as directed for the commencement of a Parliamentary survey, the bearings of all stations† visible from that point should be carefully taken in succession, right round the compass. The magnetic bearing should likewise be taken; that is, the reading of the needle, which will be the same throughout the survey, as already explained.‡ The angles should be read twice at least at each station. They may be registered in the following manner.

Angles taken at Station A, (on Turvey hill).

							Magnetic
B (Portrane)	00°	00′	00″	180°	00′	30″	85° 45′
C (Malahide Spire)	40	10	00	220	10	00	
D (Swords Round tower)	75	30	30	—	31	00	
E (Curkeen hill)	192	20	15	—	21	00	
F (Lusk Round tower)	270	00	00	—	01	10	
G (Rush windmill)	295	15	20	—	15	00	

The angles having been duly taken at A, the instrument may be removed to another station, as E, where, being properly adjusted, the vernier plate should be clamped at the reading 192° 20′ 15″, which may be called the back angle, and the telescope be directed to station A ; the lower plate must then be firmly clamped, and the cross-hairs being accurately in coincidence with the pole A, as near the bottom of it as possible, to counteract any deviation from the perpendicular, the upper or vernier plate may be released, and the telescope be directed to the surrounding stations in succession as at station A. The instrument being then removed to another station, the same proceeding is to be repeated, observing always to set the vernier to the proper back angle; thus suppose the next station were B, the vernier may be set at 360°, and the cross-hairs be fixed on station A ; or if station B has been observed from E, the reading of station B from E may be selected, and the telescope being directed to that station, the lower limb must be clamped, and the bearings of the several other stations which are visible be taken consecutively as before. The angles should be taken at as many stations as possible, and with the best instrument the circumstances will admit of; these matters will be regulated by the degree of accuracy required, or the expense to which the surveyor is limited.

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* Page 226.

† By bearing is meant the reading on the horizontal limb.

‡ Page 88.

The angles having been taken, the length of the lines calculated, and the triangulation laid down as already directed, in Section VI., the inland work may be got in, as explained in the same section.

Any sand-banks or sunken rocks that occur in the offing, should be marked, either permanently or temporarily, by buoys or beacons, and their positions fixed in the course of the triangulation; the soundings round and about them may be taken afterwards.

If the outline of a limited portion, as ten or twelve miles, of coast were required, the simplest way would be to do it by transverse surveying; the mode of executing which has been already explained in Sections V and VI. In this case the necessary land-marks and water-marks may be taken by intersections.

The lines both of high-water and low-water marks should, in general, both be shewn on the plan. High-water mark, that is the line to which ordinary spring-tides commonly rise, can generally be found without much difficulty; but in some situations the water recedes to a very considerable distance; in such cases there will be some difficulty in ascertaining the low-water line; if it be required it may be found by soundings while the tide is in. When the water does not retire too far, long offsets may be taken to low-water mark; or a number of points may be marked by beacons, and intersections taken to them, by means of which the line may be drawn in. The degree of importance attached to it, arising from the immediate objects for which the survey is to be made, will regulate the extent of time and expense which the surveyor will be justified in expending on this and other portions of his work.

It is scarcely necessary to say that the survey of a large river may be accomplished in a similar manner to that of a line of coast. The immediate objects for which the survey is to be made will regulate to what points the surveyor will direct his most vigilant attention.

Sections and Soundings.

The survey having been made the soundings may be proceeded with. The first thing to be done with reference to soundings is, to erect one or more tide-gauges. These may be constructed of the ordinary seven-inch planking, painted white, and distinctly divided into feet and inches, in a manner somewhat similar to Gravatt's levelling staff. The surveyor must be careful to have his gauge securely fixed, in such a situation as to embrace the full range of the greatest tides, from the lowest to the highest water, and so sheltered that the water may not be subject to much disturbance. If such a position cannot conveniently be found, it will be necessary to attach glass tubes to the gauges, to keep the water quiet. Major Lloyd has given a description and drawings, in the Philosophical Transactions for 1831, of gauges erected by him. The necessity for a gauge or gauges will be evident when it is recollected that all the soundings must be referred to a datum, as high-water; and that as all soundings cannot be taken at the time of high-water, means must be found to reduce them to it. This is accomplished by having a person stationed at the tide-gauge to register the height of the water at stated intervals, say every ten minutes, whilst others are engaged taking the soundings; they at the same time carefully noting the time at which each sounding is taken. By these means, it is obvious, the quantity to be added to each sounding, to reduce it to high-water, may readily be found.

Soundings not exceeding 15 feet may be taken with a staff properly graduated. For greater depths it will be more convenient to use a lead and line, duly divided and marked.*

It is manifest that the localities of all soundings must be fixed, for the purpose of shewing them on the chart. The simplest mode of managing this will be to fix a line on the shore and mark it by two or more poles, and to take the requisite number of soundings, at regular intervals, in that line produced; then have another line fixed and marked, the soundings taken as before, and so on, until as many have been obtained as may be deemed requisite.

The distances between soundings may be computed from the number of strokes of the oars; and the average distance passed over in a given number of strokes, as ten or twenty, may be found, approximately, by rowing along, parallel to the shore, in such a situation as may be measured on land.

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* It is usual to mark all soundings within the tideway in fathoms; and all above the tideway in feet. This should be noted on the plan.—See pl. vii.

Or, the boat being anchored, its position may be fixed by taking intersections with a sextant, to fixed points, as trig. stations, on the shore. These angles may be laid down by prop., page 114.

Or, two or more theodolites may be adjusted on shore, at stations previously fixed, and the bearings of the boat taken from each of them, every time she is brought to anchor.

When the soundings of a channel are to be taken, the channel should first be surveyed, at low-water; when, the surveyor being furnished with a tracing of the chart, will be enabled to locate the greater number of his soundings by sketching, at the time of taking them.

Soundings about sand-banks and rocks may be located in a similar manner. These should be observed at low-water of spring tides, as many become visible then which are not to be seen at any other time.

Mr. Stevenson, in his work before alluded to, has given ample instructions for river surveying, to which the reader is therefore referred. A plan and section (pl. viii.) being given as an example.*

The field book may be kept on the same principle as the sketch field book, No. 1, already described; using rather a large book, and so shewing as great a width as possible on one page.

The soundings may be registered in a book divided into columns, somewhat similar to a levelling field book; appropriating one column to "Time," another to "Heights," and a third to "Remarks." The mode of exhibiting the results on a chart or plan will be understood on reference to plan No. VII.

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* By the kindness of Thos. Rhodes, Esq. chief engineer, I have been allowed to extract these plans from the Shannon Report.

The soundings should be referred to some well defined and permanent mark on the shore, as the coping of a dock wall, or of a lock. In cases of importance a large stone should be properly marked, sunk in the ground, and a description of its position, and relations to the soundings be distinctly and carefully registered.

The directions and rates of currents should be shewn on a chart. The direction may be ascertained by observing the progress of a string of corks floating on the surface ; and taking the bearing of each reach of current by the compass.

The rates of currents may be ascertained by means of a log line ; or in rivers more accurately by a current meter.

APPENDIX.

TRIGONOMETRICAL SURVEY OF GREAT BRITAIN AND IRELAND.

GENERAL ROY states that so early as 1763 the government "contemplated a general survey of this island. Towards "the execution of this work, whereof the direction was to "have been committed to my charge, the map of Scotland* "was to have been made subservient, by extending the great "triangles quite to the northern extremity of the island, "and filling them in from the original map." A period of twelve years was, however, allowed to elapse, without anything being done; and then the American war broke out. The peace of 1783 being concluded, General Roy again applied himself to the subject of a general survey.

In the beginning of October of this latter year, Comte D'Adhemar, the French Ambassador, transmitted to Mr. Fox, then one of the principal Secretaries of State, a memoir of M. Cassini de Thury,† in which he sets forth the great

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[*] Made with imperfect instruments, between the years 1747 and 1755, under the auspices of the Duke of Cumberland; and in the execution of which General Roy, as Deputy Quarter-Master-General, had a considerable share.

[†] The earliest specimens of topographical maps in the west of Europe are those of France, made by Cassini and Picart. Picart measured a degree in the neighbourhood of Paris, with a view to ascertain the figure of the earth. This was repeated afterwards on the same ground by the Cassinis, and the meridian prolonged to Dunkirk, and to the Mediterranean, by a

advantage that would accrue to Astronomy, by carrying a series of triangles from the neighbourhood of London to Dover; there to be connected with those already executed in France; by which combined operations the relative situations of the two most famous observatories in Europe, Greenwich and Paris, would be more accurately ascertained than they are at present.

This memoir was transmitted to Sir Joseph Banks, then President of the Royal Society, who communicated it to General Roy, proposing at the same time, that the General should, on the part of the Society, charge himself with the execution of the operation. To this proposition the General readily assented. And His Majesty, George the Third, was pleased soon to supply the funds that were judged necessary.

This important scientific undertaking was the virtual foundation of the general surveys, not only of this kingdom, but of France, as will appear in the sequel.

The operation, the first of the kind, on any extensive scale, ever undertaken in this country, naturally divides itself into two parts. First, the choice and measurement of the base; secondly, the disposition of the triangles, whereby the base was to be connected with such parts of the English coast as are nearest to the coast of France; and the determination of their angles.

Hounslow Heath, then unenclosed, was, from its extraordinary levelness of surface, great extent, and freedom from all local obstructions, fixed upon, as the most eligible

series of triangles. The perpendicular to the meridian was also carried to some distance across the kingdom.

The school of French Engineers produced by Cassini, sent some pupils to Britain, about the year 1752, of whom Rocque surveyed Middlesex, Surrey, Berks, and other counties in England; and Armagh and Dublin, in Ireland. His pupils in Ireland form, at present, the most respectable school of land-surveyors and valuators in the latter kingdom, but they work, universally, with the circumferentor.

situation for the line to be measured. A narrow tract along the heath was consequently cleared from furze bushes and ant hills, for the purpose, by a serjeant, corporal, and 10 men of the 12th Foot.

It happened, fortunately, that a favourable line, towards the south-east, passing through a gorge of the Heath, formed by Hanworth Park and Hanworth Farm, coincided with the remarkably high spire of Bansted Church. This line, therefore, was fixed upon, and cleared to a breadth of from two to three yards. The first tracing of the base was done by means of a common telescope, held in the hand; a bell-tent being placed at the first station, near Hanworth Summer-house; and camp colours ranged in a line with the tent and Bansted spire.

STEEL CHANI.

One of the first instruments ordered from Ramsden was a steel chain, one hundred feet in length, the best that he could make.* It consisted of 100 links, each tenth link being marked with a piece of brass, like the common chain; and was, including the handles, exactly 100 feet in length. It was at first proposed to use this chain merely as a check upon the more elaborate operations with

DEAL RODS.

The bases which had been previously measured in different countries, with the greatest appearance of care and exactness, had all, or for the most part, been done with deal rods, whose lengths being originally ascertained by means of some metal standard, were, in the subsequent applications of them, corrected by the same standard. Having so many precedents to guide them, the Royal Society's officers naturally pursued the same method, on commencing the measurement of their first base, taking, however, especial care that their rods should be made of the very best materials that could be procured; with the further precaution of trussing them, to render them inflexible. They were made of an old mast of Riga red wood, of remarkably straight grain. It was at first proposed to make them of twenty-five or thirty feet in length, but such rods were found to be too unweildy; they were consequently made twenty feet in length. Plates of these rods and of all the minutiæ of their construction are also given in the works already quoted.* Three measuring rods, and a standard one were made. The stems of the measuring rods were each twenty feet three inches in length, reckoning from the extremities of the bell-metal tippings; very nearly two inches deep; and about 1¼ broad. Being trussed laterally and vertically, they were rendered, as to sense at least, inflexible. The standard was trussed laterally, only, and was made somewhat stronger, and two or three inches at each end of extra length.

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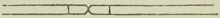
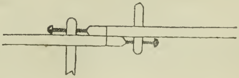
* Elaborate plates, together with a minute description of this chain, and of its several parts, are given in the Philosophical Transactions for 1785, and in "A Trigonometrical Survey of England and Wales, by Captain W. Mudge, and Mr. Isaac Dalby," published by W. Faden, London, 1799. From this latter work the following account is principally abstracted; and to it the several references apply.



The rods were so constructed that they might be applied either with their ends abutting against each other, as in fig. 1; or by the adjustment of coinciding lines. The lines were brought into coincidence by screws, as shown in sketch, fig. 2.



The latter method was first attempted, because when the rods were used in contact the errors were all one way; whereas, in the other method, the errors fell sometimes on one side, and sometimes on the other, and so destroyed each other. Two narrow pieces of ivory, each with two small screws, were inlaid into the upper surface of the rods, within 1½ inch of the extremities of the tippings. These pieces of ivory received fine black lines cut into them as in the sketch, when the lengths were laid off; and accurately determined the distance of 20 feet. Small brass wheels were arranged, with screws and springs, so as to facilitate the moving of the rods for accurate adjustment. Cross feet, about 9 inches long, were placed about 5½ inches from the ends of the rods, having their lower surfaces level with that of the stem. By means of these the rods were not only kept steady, but they served to hold vertical and horizontal brass clamps.

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* See note in p. 303.

Great care was taken to lay off, very accurately, the lengths of the rods. Mr. Ramsden prepared a beam-compass sufficiently long to take in twenty feet, trussed in all respects like the measuring rods, and fitted with proper points and micrometer. Two bell-metal cocks were screwed to one side of the standard, at about twenty feet two inches from centre to centre, and rising above the stem nearly two inches, so as to be in the same plane with the upper surface of the measuring rods, when placed upon it. A silver wire was stretched very tight along the centre of a clean, smooth, straight, and stout plank, in the same room, and six spaces of forty inches each were marked off by the side of the wire, from the Royal Society's brass standard scale,* at which points brass pins about one-tenth of an inch in diameter, were driven into the wood. During the whole of this operation the thermometer, lying by the side of the brass scale, stood steadily at 63°. A fine dot being then made on one end of the extreme pins, and wire stretched over the dot, and over the other pins, the length was re-measured with the utmost care with beam-compasses. The exact length of 20 feet, thus obtained, was taken between the points of the long beam-compass, and transferred to the tops of the bell-metal cocks, already mentioned, on the standard rod, leaving about 1½ inch in length of those cocks extending at each end, beyond the 20 feet. The measuring rods were readily marked from the standard. One of these when finished, weighed 24 pounds.

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* This scale, which is about 42 inches long, contains on it, the length of the standard yard from the Tower, that from the Exchequer, and also the French toise, together with the duplicate of the said scale.

From the extreme levelness of Hounslow Heath, the ascent from the south-east towards the north-west being little more than one foot in a thousand in the distance of five miles, it was easily seen that a curve line parallel to the surface of the sea, at that height above it, would fall very little short of the hypothenusal distance measured on, or parallel to the surface of the heath; nevertheless, for extreme accuracy's sake, it was resolved that nothing should be neglected. From the trouble and uncertainty attending the frequent use of plummets, especially in windy weather, instead of measuring base or level lines, as was usual, it was judged to be a better method to measure hypothenuses; and, having obtained the relative heights of the stations by a spirit-level, to compute the base lines. Thus it was determined that the length of this base should be obtained by measuring a line on trestles, parallel to the common surface from station to station, in equal distances of 600 feet each.

Two kinds of stands, tripods, or trestles * were used; one whose height was fixed, to be placed at the extremities of each 600 feet; and the others whose heights were adjustable according to the irregularities of the surfaces. In the

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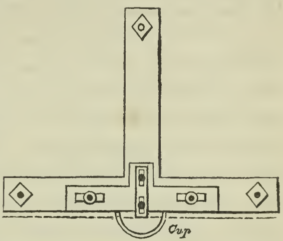
* See plate V.

middle of each of these latter an hexagonal wooden pipe descends, from the top to within two or three inches of the bottom, where it is joined by a brace reaching from each leg. This pipe received the common cheese-press wooden screw, having three sides screwed and three plane, to the top of which a square table was attached. It was embraced by a circular nut, or winch with four handles, whereby the table might be elevated or depressed; it was clamped by a flat-headed iron screw passing through one of the legs, and pressing an iron plate fixed in the inside of the pipe, against one of the plane sides of the screw. The fixed stands were 2 feet 7 inches in height; but when the glass rods were afterwards used, they had an additional 10 inches fastened on the top.

Horizontal and vertical clamps were used, by means of which the cross feet were fastened to the tables; and the rods brought to coincide by working two milled-headed screws in the horizontal clamps, against the opposite side of the cross feet.

Preparatory to measuring each hypothenuse of 600 feet a cord was stretched extremely tight along the ground, and the space divided into rod-lengths, by small wooden pins placed close by the cord, which marked the points over which the stands were to come. The stands at the extremities being then placed, and a telescope elevated to the proper line on the first, boning rods, carrying moveable vanes, were placed in the line intermediately, and the vanes being brought to the required height, rendered it easy to bring the surfaces of the stands into the same plane. A spirit level being then placed upon the table, its plane was accurately adjusted by means of wedges under the legs. It was ultimately found necessary to use deal platforms, standing on pickets driven into the ground, and properly levelled, to receive the legs of the stands.

The end of each day's work was marked by sinking into the ground a small wooden tripod, or instrument similar in shape to a T square, with three strong iron prongs, about 12 inches in length, passing through the ends of the wood; and having a brass cup situated as in the annexed figure. The cup held water in which a plummet from the end of the measuring rod was suspended. A fine line, on a brass ruler attached to the tripod, was then brought, by means of appropriate screws, to coincide with the plumb-line of gilt wire, and there clamped; and all made secure from injury or accident, and guarded by sentinels until the work was resumed.



A wooden pipe about 6 feet long and 1 foot in diameter, having an old coach-wheel firmly fixed at its lower end, was sunk at each extremity of the base. In this pipe a brass cup was placed, having a point on its cover to mark accurately the point of commencement. The wooden pipes having been found much decayed at a subsequent period, were replaced by condemned cannon.

Rough Measurement of the Base with the Chain.

On the 16th of June, 1784, the first measurement of a portion of the base was accomplished. A distance of 78 chains or 2600 yards was measured in about $3\frac{1}{2}$ hours, the mean temperature of the air being 63°. This space was re-measured next day, when the measurement fell short of the previous one by five inches. The mean temperature of this day was 65°. Five men were necessary to work the chain: two at each end, and one to adjust it, intermediately;

but subsequently a holdfast was invented to embrace the rear end, and having prongs, which being driven into the ground, kept the chain immoveable. On the second 78 chains being re-measured with the aid of the holdfast, the two measurements differed only 1½ inch. In these cases the chain was used on the surface of the ground.

A transit instrument was now adjusted over the first picket, when it was found that the advanced picket was not truly in a line with Bansted spire; the transit was therefore moved a few inches until the three points were found to be in the same vertical plane. The first picket was then moved and placed exactly under the axis of the telescope. The same operation was repeated at two intermediate stations, and at the north-west extremity of the base, when it was found that the tract cleared by the soldiers was, at one point, five feet out of the right line.

The lower end of the base was found to be 36.1 feet above the surface of the Thames at Hampton.

MEASUREMENT OF THE BASE WITH THE DEAL RODS.

On the 16th July, General Roy, accompanied by Sir Jos. Banks, Messrs Blagden, Cavendish, Lloyd, and Smeaton, commenced the measurement with the Deal Rods; when five hours were consumed in measuring 300 feet. This rate of progress was deemed so tedious that it was resolved to discontinue the mode of measurement by coincidences, and to proceed by the method of contacts. The rods being placed in contact with each other the measurement was advanced on the ensuing day to the middle of the fourth hypothenuse.

Having proceeded in this way for some days, duly comparing the rods with the standard, it was no small disappointment to find that they were so liable to lengthen and shorten, by the humid and dry states of the atmosphere, as to leave no hope of being able, by their means, to determine the length of the base to that degree of precision which was aimed at. But it was judged proper to proceed with the measurement. The rods on comparison were found, not unfrequently, to exceed the standard measure by one-fifteenth of an inch.

The measurement of the base, of 27400 feet, with the rods, exceeded that with the chain by two inches and nine-tenths only. Nevertheless, it was found, by subsequent experiments,* that not only the measuring rods, but the standard itself, however carefully kept, was considerably affected by the state of the atmosphere; so much so as to render it improper ever to have recourseto them for a like purpose again. In consequence of these discoveries it was resolved, at the recommendation of Lieut.-Colonel Calderwood, to make trial of glass rods.

Description of the Glass Rods.

It was first proposed to have solid rods, but it was found to be impossible to take, at once, a sufficient quantity of the melted metal on the irons to make them solid, of the required dimensions. Glass tubes were therefore prepared, and enclosed in deal cases very carefully constructed, eight inches in depth, of the same width in the middle, and tapered off towards each end to 2¼ inches. Each tube was made fast, in the middle, to its case, by wrapping it round with packthread, saturating that with thin glue, and then forcing on all a strong mahogany collar. They were also supported at two other points, as well as at the extremities. Very good corks were nicely fitted to the bore; a brass tube was placed in the middle of the cork, which received a steel pin formed to screw into the inner end of the brass tube.

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* See p. 314.

The steel pin carried outwardly a button and neck of bell-metal. The whole was made exactly 20 feet in length when in adjustment. These tubes were carefully and ingeniously furnished with slides, wheels, &c. so as to be used with accuracy and facility. Two thermometers were attached to each tube. Plates and descriptions in detail of them, and of their several parts, are given in the works before referred to, when describing the deal rods. Each tube or rod, complete, weighed about sixty-one pounds. They were constructed by Ramsden.

Re-measurement of the Base.

It being foreseen that the measurement with deal rods must be entirely rejected, it was now resolved upon to make a double measurement at once; one with the chain, and the other with the glass rods, on the same stands, the chain to be placed twice in advance, and the rods to follow in succession. Coffers, each nearly 20 feet in length, were accordingly prepared, so that the chain might be applied in the same plane as the rods. Five of these coffers made about 98 feet; thus leaving a space between each group or series, for the middle or slide stand, on which the handles of the chain were supported, and which consequently received the mark made at the end of each hundred feet. Thus each 98 feet of coffering was supported by six stands, the intermediate ones, nor the coffers, not even touching the central or slide stands. To the hinder end of the chain, a flat steel rod, about two feet in length, was attached; it was inserted into a small wooden frame which slipped on to the top of the stand immediately behind the chain. Holes were pierced in the rod to admit a steel pin, which was passed through it into a hole in the wooden frame. The stand to which this was attached was double weighted. The end of the steel rod nearest to the chain was formed into a screw, and received on it a forked hook, fitted to hold the handle of the chain. Within the forked hook a milled-headed nut worked, for bringing the chain to coincide with its point of commencement. For the leading end, a frame carrying a pulley was provided. A rope passed over the pulley, having a weight of fourteen pounds suspended at one end of it; whilst a forked hook at the other end laid hold of the handle of the chain. By means of these two apparatuses the chain was always kept to the same degree of tension in its coffers; in each of which a thermometer was placed; the whole being covered by a piece of linen cloth. The coffers were above five inches deep, the sides being nailed at the middle to the intersole or bottom, forming a section like the letter H.

All things being in readiness the double measurement was commenced. In the course of the first day a distance of 1000 feet was measured; when it was found, that the line marking the extremity of the chain measurement, fell short of the line denoting the end of the glass rod measurement, by just two-tenths of an inch. Which difference, taking into account the contractions due to the glass and the steel, should have been 0.17938 of an inch, to have made the two results exactly agree; being a real difference of only 0.02060 of an inch. Supposing the whole distance to have been measured with the like success, this would have given a defect of something more than half an inch, on the whole length of the base.

Although this result was so very satisfactory, yet from the, so called, tediousness of the operation, and the advanced season of the year, it was deemed prudent to give up, for the present, the further measurement with the chain, and to complete the work with the glass tubes.

On the 30th of August the measurement with the glass rods was completed; when the extremity of the 1370th rod overshot the centre of the pipe terminating the base towards the south-east by 1.49 foot. Hence when the several equations for expansions were respectively taken into the account, it was found, that the total alteration of the deal rods from the humidity of the air, had been over-rated by 20.964 inches.* General Roy was of opinion, that the chief part of this difference proceeded from over-rated expansion; that is to say, the rods, when brought into use, contracted sooner than had been imagined, and thereby gave a shorter measure than what was assignable to them from the mean of any two or more comparisons.†

MICROSCOPIC PYROMETER *used for determining the expansion of the metals concerned in the measurement of the Base.*

A minute description, and elaborate plates, are given, in the works already mentioned, of the Pyrometer, its microscopes, &c. invented and made by Ramsden; and used to obtain the equations for the expansions of the standard scale, steel chain, and glass rods, applied to the apparent measurement of the base. It was adapted for testing any rods of five feet or less, in length, to $\frac{1}{64000}$ part of an inch. It consisted of a strong frame, having firmly screwed on the top of it two deal troughs, upwards of 5 feet in length; each of these troughs, which was about 3 inches square in the inside, contained a cast iron prism 1¼ inch square. One end of each prism was screwed down to the brass pieces on which they rested, so as to be immoveable with regard to their troughs; the other ends were kept easy, but steady, in their collars. Between the deal troughs, one of copper, as a boiler, was placed. Twelve lamps were used to bring the water in the copper to boil. It was found, by burning oil in the lamps, the heat of the water could not be raised about 209° or 210°, but with spirits of wine it was brought into violent ebullition.

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* See p. 311.

† Trig. Survey, Vol. 1. p. 69.

EXPERIMENTS WITH THE PYROMETER.

To fill the troughs completely it required from 25 to 30 pounds of pounded ice, which was always put in with great care, so as to apply it as compactly as possible to the standard prisms and rod respectively, with but little water added. It was found that the water drained from the ice itself, was that which should properly be made use of, in order to bring the whole to the true freezing temperature; if common water was used the thermometer kept always half, and sometimes three quarters of a degree above 32°. A quarter of an hour was found to be more than sufficient to give to all the freezing temperature; as well as to render the lens on the expanding rod stationary, after the water supplying the place of the ice had been brought fairly to boil.

The pyrometer having been adjusted, by giving sufficient time for the standard prisms and rod to contract to the true freezing temperature, as was easily known by the micrometer wires becoming stationary with regard to the marks, the ice was then removed from the copper trough; and the same being filled with water nearly on the boil, the ebullition was completed, and kept up by means of the lamps now lighted. The expansion answering to the 180° between freezing and boiling, was now measured by working the micrometer screw until the bisection of its wire with those of the mark was again complete. The whole expansion being then divided by 180° gave 0.0075 as the expansion or contraction for one foot, due to 1° of change of temperature.

ULTIMATE DETERMINATION OF THE LENGTH OF THE BASE ON HOUNSLOW HEATH.

The hypothenusal length of the base was 27402.8204 feet. This being reduced to the level of its south-east extremity; augmented by the excess of the expansion above

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the contraction of the glass rods, reduced to 62°, as had been usual in former operations of the same nature; augmented also by the equation for 6°, difference of temperature of the standard brass scale and the glass rods, between 62° and 68°, this last being the heat in which the lengths of the glass rods were laid off; and reduced from the height of the lowermost end of the base above the mean level of the sea,* supposed to be 54 feet; the true ultimate length of the base, reduced to the level of the sea, and making a portion of the mean circumference of the earth, became 27404.0137 feet.† One third of the rise of spring tides was assumed as the mean level of the sea.‡

A table is given of the expansions of metals, deduced from the experiments made with the microscopic pyrometer.

THE TRIGONOMETRICAL OPERATION, WHEREBY THE DISTANCE BETWEEN THE MERIDIANS OF THE ROYAL OBSERVATORIES OF GREENWICH AND PARIS WAS DETERMINED.

In the spring of 1787, Sir Joseph Banks opened a correspondence with the Academy of Sciences, regarding the cooperation expected on their part for connecting the English and French triangulations. Dr. Blagden had engaged to assist, on the appointment of the Royal Society, and the Comte de Cassini was appointed by the Academy of Sciences, to superintend their part of the business.

Mr. Ramsden having completed the instrument intended for the measurement of the angles, the observations were commenced on the 31st July, at the station near Hampton Poorhouse. In September the instrument and the various parts of the apparatus were removed to Dover, at which place Messrs. de Cassini, Mechain, and Le Gendre arrived on the 23rd. In the course of two days everything was settled in the most amicable manner possible, with regard to the times of reciprocal observation.

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* The amount of this reduction was only 0.0706 of a foot.

† This line was re-measured, in 1791, with a steel chain, when the length was found to be 27404.32 feet; the difference between the two measurements therefore was 0.31 of a foot.

‡ See page 199.

A great number of white lights, fitted for long distances, and a reverbatory lamp, being supplied to the French gentlemen, they departed for Calais, accompanied by Dr. Blagden, who attended them during the time of co-operation, until the 17th of October. The weather, for the greater part of the time was extremely bad, nevertheless observations were fortunately made at Dover and Fairlight Down, from whence the two distant points, on the French coast, of Blancnez and Montlambert, were intersected with great accuracy, and the triangular connection between the two countries thereby established.

It was found to be impossible, from the tempestuous state of the weather, to complete the necessary observations, until the following spring.

During the summers of 1786 and 1787 a survey was made of part of Romney Marsh; on which it was intended to measure a base of verification.

APPARATUS MADE USE OF, AND MEASUREMENT OF THE BASE OF VERIFICATION IN ROMNEY MARSH.

Having ascertained, on Hounslow Heath, how very accurately distances might be determined by the steel chain, it was resolved on to apply it to the mensuration of the base in Romney Marsh, particularly as it was feared the glass rods might be broken in transporting them from London; besides the measurement with the chain would be accomplished with much greater facility.

Accordingly a number of beech posts, three inches in diameter, and from two feet six inches to about four feet in length, were provided. They were shod with iron, and each had a cast iron ferule, with two dove-tails projecting from it;* these posts were driven into the ground having the dove-tails as nearly as possible in the direction of the base. They stood twenty feet apart, except those at the extremities of the chain; here, as with the glass rods, one post, called the central post, was placed wholly unconnected with the coffering apparatus, on which each chain's length was marked. Each of these central posts carried on its surface a brass scale, of six inches in length, moveable in a slide, by a finger-screw. The nearest division on this scale being brought to coincide with the end of the chain, was registered, together with the temperature of the chain, as shewn by a thermometer in each coffer. The chain was then moved forward.

Deal Coffers.

Fifteen deal coffers, numbered from 1 to 15, were necessary for the length of three chains. Six of them, that is to say, the first and fifth, &c. being the first and last of each chain, were only 19 feet 4 inches in length. The other nine being of the full 20 feet. They resembled in shape, and nearly in dimensions, the cases of the glass rods. Nine or ten inches of the under extremities of the cheeks or sides were cut off, so as to permit the bottom or intersole itself to rest on the irons.

The surface of Romney Marsh having been formerly covered by the sea, was so level as not to require levelling, except for a short distance at each end of the base.

Result of the Measurement.

The total apparent length of the base, as given directly by the chain, was 285.36736 chains = 28536 feet, 8.835

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* These were afterwards replaced by others made of hammered iron, as those of cast-iron were burst by driving the posts into the ground.

inches. This distance was reduced, for the contraction due to the mean temperature, by 12.8 inches; for the correction of two hypothenusal distances at one end of the base, by 3.023 inches; and for the height above the mean level of the sea, supposed to be 15½ feet, by 0.166 of an inch; and the sum increased for half the ascertained wear of the chain, by 3.282 inches. The ultimate length of the base in the temperature of 62° of Fahrenheit was found to be 28535 feet 8.128 inches

To prove satisfactorily that the expansion of the chain was accurately determined, it was usual, at the close of each day's work, to leave the scales marking the extremities of the last chain upon their respective posts until next morning. On resuming operations, the chain being applied anew to the scales, if the temperature continued the same, the coincidences were found to be equally accurate as on the preceding night; but if it differed one or two degrees the chain never failed unequivocally to show it.

The great Theodolite, with which the angles in the previous trigonometrical operation had been observed, was made by Ramsden. The circle or limb was of brass, and was three feet in diameter.* It had only two verniers; but was constructed with great care and accuracy, and is fully described and illustrated in the works already referred to. It weighed in the whole about 200 lbs. The stand, steps, stools, tent, &c. weighed at least as much more.

A portable scaffold was constructed for the purpose of raising the instrument above the surface of the ground. It consisted of an inner scaffold for supporting the instrument, and an outward one for the observers, wholly free of each other. These scaffolds, both outward and inward, were divided horizontally into two parts; so that the upper part only might be used when it became unnecessary to raise the instrument to a greater height than 15 or 16 feet. The whole being 30 feet in height. A three-sided trough or trunk, which turned easily on its axis, protected the silk thread that suspended the plummet.

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* It was found that the effects of heat and cold on the limb would produce considerable errors, if it was subjected to streams of air passing over its surface. This was avoided by keeping up the walls of the tent, in windy weather, leaving only a sufficient opening for the observation; and at other times throwing the external tent entirely down; and so maintaining an equal temperature.

Tripod Ladder.

A tripod ladder about 35 feet in height, was contrived, which carried on its top either a globe lamp, a white light, or a flag-staff, as occasion required. A heavy plummet denoted its centre point on the ground.

Portable Crane.

A portable crane was constructed, for weighing up the instrument to the tops of towers or other buildings.

Stations.

The several stations were distinguished, as far as possible, by permanent marks; by means of which the observations might be repeated at any future time. These marks are respectively described in the detailed account.

Spherical Excess.

Since the sum of three angles of a spherical triangle is greater than two right angles, it follows that the sum of the three observed angles of each of the triangles of this survey ought to exceed 180°. In the detailed account, tables of the triangles are given, which contain one column showing the " spherical excess" by computation, and another containing the "difference or error" by observation, of the angles. In these triangles, where the lengths of the sides varied between five and fifteen miles, the figures in the column of " spherical excess" seldom differed from those in the adjoining column headed " difference or error" more than 1″. The sum of the three angles was, however, sometimes more and sometimes less than 180°.*

The length of the base on Romney Marsh, by calculation, was found to be 28533.3 feet, by measurement 28535.66 feet; the difference therefore is about 2 feet 4 inches. A mean between these distances was made use of in the computations.

Dovor Castle was the only station on the English side of the water to which the French academicians made observations. From the English side stations were observed at Montlambert, Blancnez, and Calais. The following table affords a comparison of the results.

	By the Eng. triang.	By the French triang.	Diff.
Dovor from Montlambert feet	168827	168806	21
Dovor from Blancnez - -	116660	116648	12
Dovor from Calais - - -	137455	137442	13
Montlambert from Blancnez -	77238	77228	10

But if the distances were determined from the base on Hownslow Heath *only*, the differences would be 14, 7, 7, 6, feet respectively.

OPERATIONS AT GREENWICH OBSERVATORY.

The great theodolite was raised on a temporary scaffold, to a height of 38 feet above the floor of the transit-room of the Observatory; and the angles between the south meridian mark, and several surrounding objects, accurately measured. A table is given, containing the bearings of the Trigonometrical stations, from the parallels to the Meridian of Greenwich; also their distances from the first meridian, and its perpendicular.

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* Vide page 318. p 275

The Meridian of Paris, is distant from the Meridian of Greenwich, at Dunkirk, 537,978 feet, on a parallel to the perpendicular taken at Greenwich.

The parallel of latitude of Paris, was computed to be distant from the parallel of Greenwich 160,659 fathoms.

And the length of the degree, on the meridian, in lat. 50° 10′ (the middle latitude, nearly, between Paris and Greenwich) was computed to be 60,843 fathoms.

The result of the observations and calculations was, that the latitudes of Paris, Dunkirk, and Greenwich are 48° 50′ 15″, 51° 2′ 11″ 4, and 51° 28′ 40″ respectively. But taking these results as data, it would appear that the difference in the lengths of the meridional degrees in latitudes 49° 57′ and 51° 16′ is about 135 fathoms; which will not accord with any of the received hypotheses for determining the figure of the earth.* Hence it would appear, notwithstanding all the care taken on both sides of the Channel, that some error has crept in.

The heights of the stations, above low-water, were ascertained by angles of elevation and depression; both angles were observed when that was practicable. Demonstrations are given of the rules by which the refractions were calculated.

In the course of these operations the situation of a number of conspicuous, and well defined objects, was determined, "for the improvement of the maps of the country, and the plan of the city of London and its environs." A diagram is given of these secondary triangles; together with tables of the angles, and the distances. Tables of latitudes and longitudes are also given.

After the death of General Roy a considerable time was allowed to elapse without any apparent intention of renewing the business of the Survey; when a casual opportunity presented itself to the Duke of Richmond, then Master General of the Ordnance, of purchasing a very fine instrument, made by Ramsden, of similar construction to that used by General Roy; as also two hundred-feet steel chains,* by the same artist. These, comparatively trifling circumstances, gave the matter a new stimulus, and his Grace, with his Majesty's approbation, immediately gave directions to prepare all the necessary apparatus.

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* Trig. Survey, p. 163.

One of the first operations was to find the relative lengths of the new chains. This was accomplished by first stretching one of them in a set of coffers, carefully adjusted for the purpose, with a thermometer in each coffer; securing one end of the chain by pickets, and suspending from the other end a weight of 56 lbs.; then allowing the whole to remain till all the thermometers stood steadily at the same height. The ends of the chain being then in perfect coincidence with particular divisions on the brass register-heads, the chain was quickly taken out, and replaced by the other; by which means the difference of their lengths was measured with the micrometer; the thermometers remaining steady during, and after, the operation. The difference of the lengths amounted to only $\frac{6}{260}$ths of an inch. One of the chains being kept as a standard, whilst the base on Hounslow Heath was re-measured with the other, it was found that the measuring chain lengthened one-26th of an inch in measuring 236 chains.

For finding the Rate of Expansion.

The chain being placed in the coffers as described in the last article, the whole was covered with a linen cloth, and allowed to remain until the thermometers denoted nearly the same temperature; when a perfect coincidence was made on the register heads, at each end of the chain, and the thermometers noted. Every thing remained in this state until the coincidence at the weight end of the chain was observed to be altered, and the thermometers nearly the same height; at which instant, they were again read off, and the alteration of the chain measured with the micrometer. This operation being repeated the mean results of nine experiments was found to be 0.0075 of an inch to 1° of Fahrenheit, on 100 feet of blistered steel; which differed only $\frac{13}{100000}$th parts of an inch from General Roy's conclusion with the pyrometer; but the number .0075 was preferred as being deduced from experiments made with the chain itself.

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* These chains contained but 40 links, each of 2½ feet long.

Commencement of the General Trigonometrical Survey, as such, 15th Aug. 1791.

It was found expedient to place the coffers upon trestles, such as had been made use of by General Roy, with the glass rods; Sir Joseph Banks, therefore, obligingly lent the same trestles.

It having been determined, in the first place, to measure the base on Hounslow Heath, operations were commenced on the 15th August. During the measurement the alterations of temperature were carefully noted, the thermometers usually remaining in the coffers from 7 to 15 minutes, according to circumstances; when the sky was much overcast, a shorter time was generally found to be sufficient. A table is given of the details of each day's work; by which it appears that the hypothenuses were of different lengths; varying from 1 to 34 chains.

Dr. Hutton, Dr. Maskelyne, astronomer royal, and Mr. Ramsden, were present at the termination of the measurement, on the 28th Sept. When, Mr. Ramsden having determined, by his brass standard scale, and beam compasses, that the horizontal distance from the end of the last chain to the axis of the pipe, was 21.055 inches; it appeared that the length of the base was 274 chains and 21.055 inches. This apparent length being reduced to the plane of the horizon, and having undergone the necessary corrections for wear of chain, and change of temperature, was found to exceed, by about 2¾ inches, the measurement with the glass rods; the mean of the two results was therefore taken as the true length.

THE ASCERTAINING OF THE ACTUAL LENGTH OF THE CHAINS.

Mr. Ramsden having found that the plank, to which he had transferred points from his brass standard, was liable to a daily expansion and contraction, determined to adopt some less exceptionable method of measuring the lengths of the chains. Considering that the expansion of cast iron is nearly the same as that of steel, he procured a prismatic bar of that metal, of 21 feet in length, on which to establish a permanent standard. Mr. Ramsden set his great plank on edge, duly prepared with brackets to support the bar, with one of its angles downwards, so that one of the faces of the prism became the upper surface. On this upper surface of the bar he then laid off a length of 20 feet* in the temperature of 54°. The chains were then measured in twenty-feet lengths; being kept straight, on rollers, by a weight of 56 lbs.

COMMENCEMENT OF THE TRIGONOMETRICAL OPERATION.

Having re-measured the base on Hounslow Heath, it was determined by Captain Mudge, R. E., and Mr. Isaac Dalby, to whom the charge of conducting the operations was entrusted, to proceed with a series of triangles immediately southward; to be connected with General Roy's series: that so an early proof might be obtained of what degree of accuracy had attended both operations. Several of General Roy's stations were therefore visited early in 1792. The Duke of Richmond gave instructions that, in choosing stations those on towers and high buildings were to be avoided, as getting an instrument on them had been found both difficult and dangerous.

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* See p. 319.

Lamps, for using at night, were constructed, enclosed in strong tin cases, having plates of ground glass in front. Two of these lamps were of 12 inches diameter, and a third of 22 inches; the last of these could be clearly distinguished at a distance of 30 miles. These lamps were found to be far preferable to white lights, when the distance admitted of their use; because observations to the lights could not be repeated without burning other lights, and those could not always be observed at the necessary moment.

Observations in day-light were generally made either in the morning or the evening, when the air was free from vapour, and without that quivering motion, which, in summer, is so usual, in the middle of the day.

METHOD OF OBTAINING THE DIRECTION OF THE MERIDIAN.

A lamp was placed at a station about six miles distant from the place of observation, for the purpose of referring the pole-star to it. Double azimuths were taken to the star, at its greatest eastern and western elongations. Great care was taken to have the instrument ready, before the star came to its greatest elongation; but, as, four minutes either before or after those periods, the star moves only about one second in azimuth, the time was shown sufficiently near by a good pocket watch. The star was subsequently observed from different stations.

Tables are given of the angles of the several triangles. Descriptions of the marks left at several of the stations are also given.

MEASUREMENT OF THE BASE OF VERIFICATION ON SALISBURY PLAIN, IN 1794.

It had been found advisable to have the pickets placed very accurately in the line to be measured, and also to have the register heads exactly in the plane of each hypothenuse. An arrangement was therefore effected by which the transit instrument itself was used as a boning telescope. The angles made by the hypothenusal lines with the horizon, were likewise measured with the same instrument. It was found necessary, frequently, to step the commencement of one hypothenuse up or down from the termination of its preceding one, in this base. This was carefully accomplished by a plummet suspended by a fine silver wire.

The apparent length of this base was 366 chains. Its measurement occupied seven weeks. The measuring chain, on comparison with the standard, was found to have lengthened, during the operation, only one-260th of an inch. Having undergone the necessary corrections, the horizontal length of this line, at the level of the sea, was found to be 36574.4 feet; or nearly seven miles.

The angles and sides of seventeen triangles are given, by means of which the length of the Base of Verification was deduced, from that on Hounslow Heath. Five different results are obtained, but the one which was deemed most unexceptionable was 36574.7 feet. Thus differing from the measured distance by little more than three and a half inches in the seven miles.

KING'S SEDGEMOOR BASE.

It is to be regretted that no ground further westward could be found sufficiently favourable for the measurement of another base of verification; but neither in Devonshire nor in Cornwall could any fit place be found. King's Sedgemoor, in Somersetshire, was therefore ultimately selected, as the most advisable site for the measurement of a new base.

As there were several very large ditches, or drains, to be crossed in the measurement of the proposed line, it was deemed advisable to be provided with a chain of 50 feet in length; as the handles of the 100-feet chain would often have had their places in ditches; this chain was found to be very serviceable. The same apparatus was made use of as on the measurement of the other bases of verification. On arriving at the extremity of the 114th chain, it was found necessary to diverge from the right line, in consequence of a great ditch occurring coincident with the base; this was accomplished by placing the great theodolite in the line, and going off at right angles. It is not so stated, but of course the instrument was then placed over the end of the offset, and another right angle measured, and then the measurement carried on parallel to the base line,* for 19 chains' length, at the end of which the same operations were repeated, and the original direction resumed. King's Sedgemoor being sufficiently level, this base was measured horizontally; but the ground was so soft and shaky that those who attended the handles of the chains either used long stools, or placed themselves so as to divide the pressure arising from the weights of their bodies, equally on each side of the pickets. Operations commenced on the 11th July and terminated on the 16th August, 1798.

A diligent search was again made after some spot in Cornwall, for a base of even two or three miles in length, but the search proved fruitless.

The horizontal length of the Sedgemoor base was 27680.14 feet, or nearly 5¼ miles. It was supposed that the total error was not less than six, nor more than nine, inches.

Refraction.

In the course of this year's operations some pains were bestowed to ascertain the refractive power of the air. These observations tended to prove, that when rays of light pass horizontally, although considerably distant from the surface of the earth, they are less bent or refracted from their rectilinear courses, than theory and opinion had laid down as fact. It is very certain, however, that objection lies against particular conclusions drawn from such *data* as we possess; because the angles of elevation and depression of corresponding stations are observed *at different times*, and therefore, almost always, under different circumstances.

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* See page 131.

The air is not often clear enough, nor sufficiently free from tremulous motions, for these delicate observations; but due care being taken, it appears that considerable accuracy may be arrived at. Observations were made at seven or eight different hours of the day; the results sometimes differed several seconds from each other. Thus at Arbury Hill, Brill at noon read 11′ 15″; and at 4 p. m. on the same day the reading was 10′ 5″.

The observations for refraction were made, experimentally, at two corresponding stations simultaneously. On one occasion the change in the observed angle of elevation at Whiteham Hill station from one hour to another was 44 seconds, *without* a corresponding change at the other (White Horse Hill) station; thus proving that some *partial* alteration, from floating strata, had taken place in the refraction, near the former station; suggesting, that to afford any accurate conclusions, a long series of observations would be necessary; and most probably, that perfect accuracy could never be ensured.

Barometers were observed at the several stations, but in some instances the wind was so high, and the stations so distant from each other, it was not to be expected that a true result should be obtained. The differences of altitudes of the stations constituting the extremities of the two most favourable arcs, obtained by means of the observed angles of elevation and depression, as well as from the heights of the mercury in the barometer, will be as follows.—

Arcs.	Obs. Ang.	Barom.	Diff.
1	317 feet	282 feet	35
2	60 —	15 —	45*

MERIDIONAL ARC.

It was resolved that the operations carried on in 1800, 1801, and 1802, should be made subservient to the purpose of measuring a portion of the meridional arc, running from Dunnose, in the Isle of Wight, into the northern part of Yorkshire. Mr. Ramsden had commenced a zenith sector to be used on this occasion, but died before its completion; consequently it was finished by Mr. Berge, in the spring of 1802.

Major Rudge determined to measure a portion of the meridian which proceeds from Dunnose, to the mouth of the Tees; because he had reason to suppose it the longest meridional arc in Britain, free from any apparent obstruction, from considerable hills or otherwise. He was led to select Dunnose for one of its extremities because he had then the means of ascertaining the distance of the Royal Observatory from the northern or southern end of his line, and consequently of connecting it with the parallels of Dunkirk and Paris; and of making any corrections of latitudes previously ascertained.

Dunnose being fixed on, such stations were selected as would give the sides of triangles, carried on as nearly as possible in the direction of its meridian, of sufficient length, and properly inclined to it. The northern extremity was fixed as near the meridian of the southern one as possible; and likewise in the neighbourhood of Misterton Carr, near Clifton, a small village in the vicinity of Doncaster, in the northern part of Lincolnshire, which afforded a plane of sufficient extent for a base.

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* Vol. II. page 190.

In 1800 and 1801, the angles of the triangles constituting the meridional series, amongst others, were observed; and in the latter year, the new base was measured. While the instrument was at Clifton the direction of the meridian was obtained from numerous observations on the pole-star; similar observations having been previously made at the station on Dunnose; and subsequently at Arbury Hill, near Daventry, a station almost in the middle of the arc.

An elaborate description, and plates, of the zenith sector are given, with Ramsden's improvements, and the several adjustments; together with the means taken to ensure accuracy of observation.

MISTERTON CARR BASE LINE, MEASURED IN 1801.

The apparatus used for the measurement of the base, was the same as that previously employed; and the like pains were taken to obtain accuracy. Two large blocks of oak, with square holes on their upper surface, were sunk in the ground at the extremities of the base; the point of intersection of the diagonals of each hole severally denoting them. These diagonals were drawn on lead, cast into the holes, and ground to a smooth plain, even with the surface of the block.

On comparing the chains, with the usual care, it was found that the differences were, as nearly as possible, the same as they had been observed at Sedgemoor.

The measurement of this, the fourth base, commenced on the 6th of June; and was continued without interruption from bad weather or other causes, till the 28th of July, when it was concluded with the 263rd chain, the overplus 38.321 feet, being carefully determined. The wear of the chain, during the operation, was found to have been exactly one-260th part of an inch.

After making the requisite corrections for temperature, difference of chains, &c. the true length of the base, in the temperature of 62°, was 26342.712 feet, or nearly 5 miles; which Major Rudge thought could not exceed nor fall short of the truth by more than two inches. This line was measured horizontally.

Subsequent to the measurement of this base, the chains were both remeasured by Mr. Berge; when the measurement of the standard differed from that by Ramsden something less than $\frac{2}{100}$ of an inch; the length of the working chain came within $\frac{5}{1000}$ of an inch of the former measurement, allowing for the wear.

Major Rudge observes that "in carrying on a series of triangles, whether for the purpose of a meridional measurement or otherwise, it is proper that a base of verification, answering at the same time as a new one of departure, should be measured every 100 miles at least. With this idea, therefore, the foregoing triangles, as well as those composing the remaining part of the series should be furnished with three base lines, viz. one at each extremity, and the other in the middle. In calculating the sides, were the series thus circumstanced, it would be right to depend on each base for one-third of the distance between it and the one next at hand, and use the mean result, as derived from the two adjoining bases, for the true lengths of the several sides within the other third. Thus if two bases were found at the extremities of the arc in question, and one in the middle, as about Brill; the computation should be carried on, from the extreme bases, about one-sixth part of the meridional distance; and from the middle base, one-third of the intermediate distance on each side; the remaining two arcs being determined from the respective base lines."

The result, or length of the meridional arc, found by these proceedings was as follows, viz. 1,036,334.4 feet = 196.27 miles, the distance between Clifton and the perpendicular to the meridian of Dunnose; which may be taken for the true length of the arc itself, as the distance of the former station from the meridian of the latter is only 4770 feet.

The length from Dunnose to the perpendicular to the meridian of Clifton was found to be 1,036,333.9 feet; making a difference of only half a foot between the two results. Three feet were to be added, because the zenith sector was not set up exactly over either of the permanent station-marks; the total length of the arc of the meridian, between the two stations, was therefore 1,036,337 feet.

Elaborate details of observations, and the deductions therefrom are given, which it is not deemed necessary to repeat here, as it is presumed sufficient for general information has been given, and those who require the details will prefer referring to the original work.

From these operations it appears, that the length of a degree on the meridian, in latitude 52° 2′ 20″, is 60820 fathoms. This conclusion is deduced from the supposition of the whole arc subtending an angle of 2° 50′ 23″ 38 in the heavens, and a distance of 1,036,337 feet on the surface of the earth. But this, coupled with the consequent deductions, does not accord with the supposition that the earth is an ellipsoid, with the ratio of its axes as 229 to 230. If the measurement of the terrestrial arc be sufficiently correct, and the earth of an elliptical form in these latitudes, either the arcs affording the deductions are incorrect, or some material deflection of the plumb-line took place, at one or two stations, from the effect of attraction; indeed, the general tenor of the observations seems to prove, that the plumb-line of the sector was drawn towards the south at all the stations; and that by attractive forces which increased as they proceeded northward.

The third and last volume, published in a separate form, of an "Account of the Trigonometrical Survey," &c. contains a large mass of topographical matter, which, combined with that given in the accounts previously published, comprises the trigonometrical survey of almost all England, and the whole of Wales, with a part of Scotland.

The first portions of the detailed survey, as of Kent, Essex, and part of Surrey and of Sussex, were made by Mr. Gardner, chief draughtsman at the Tower of London, and other civil assistants. The regular survey, performed by the Royal Military Surveyors, was commenced in Somersetshire, in the year 1804.

RHUDDLAN MARSH BASE.

The fifth and last base was measured in Rhuddlan Marsh, North-Wales, in the year 1806. It was measured with the same care and the same apparatus as the previous lines. Its apparent length was 24512.208 feet; and this being reduced for wear of chain, temperature, &c. the true length of the base in the temperature of 62° was 24514.26 feet. In the course of the subsequent operations it appeared, that if the computation had been carried on directly from the base measured on Misterton Carr to Rhuddlan Marsh, the measured and computed lengths of the latter base, would not have differed more than 9½ inches.

MERIDIONAL ARC.

A meridional arc was computed between the parallels of Dunnose in the Isle of Wight, and Delamere Forest, the details of which are given. The result, which was deemed sufficiently consistent and satisfactory, gave 60823 fathoms in latitude 52° 34′, or the centre of England, as the length of one degree.

No official account of the subsequent operations has yet been published, but a memoir of the Irish Survey is in preparation, which will have some reference to the subject. It was found to be impossible to penetrate the barrier of official mystery maintained by the officers in charge of the work: it can therefore be stated in general terms only, that the portion of England yet to be done will be laid down to a scale of six inches to the mile; this will include the northern and eastern counties. The whole of Scotland will be laid down to the same scale.

IRELAND.

In the latter end of the year 1823 stations were observed on the coast of Ireland, from the coast of Scotland, and from the Western Islands. And in the year 1825 the regular survey of Ireland was commenced, in the northern counties. In 1828 and 1829 a base line was measured in the county of Londonderry, some particulars of which have already been detailed in this volume. The survey is now (1842) nearly finished, but no other base has been measured. The Memoir which is in course of preparation, will doubtless furnish much important information. The plans are being engraved to a scale of six inches to the statute mile, and may be purchased by any person at a very moderate price.

This survey has been executed under the direction of the Master General, and Board of Ordnance.

THE DOOMSDAY BOOK.

This, one of the most ancient records in the kingdom, was known by different names, as "Rotulus Wintoniæ," "Liber Regis," &c. Stow (Annals, p. 18) says, "The Book of Bermondsey says this book was laid up in the King's Treasurie (which was in the Church of Winchester or Westminster) in a place called Domus Dei, or God's House, and so the name of the book." The author of "Dialogus de Scaccario" says, "Hic liber ab indigenis *Domus Dei* nuncupatur, id est Dies Judicii, per metaphoram," &c. And this latter derivation has also been adopted by Rudborne and other authors.

The precise date of Doomsday book is likewise a matter of uncertainty; the authorities differing between 1083 and 1087. It appears, however, that this survey was executed in a very short time, and most probably was completed in 1086.

For its adjustment, certain Commissioners, called the King's Justiciaries, were appointed. Those for the Midland counties, if not for all the districts, were, Remigius Bishop of Lincoln, Walter Giffard Earl of Buckingham, Henry de Ferers, and Adam the brother of Eudo Dapifer; who probably associated with them some principal persons in each shire. This appears from a curious document in the register of St. Mary, Worcester, usually called Hemings Chartulary, preserved among the Cotton MSS.

The Inquisitors, upon the oaths of the Sheriffs, the lords of each manor, the presbyters of every church, the reves of every hundred, the bailiffs, and six villains of every village, were, it appears, to inquire into the name of the place, who held it in the time of King Edward, who was the present possessor, how many hides in the manor, how many carucates in demesne, how many homagers, how many villains, how many cotarii, how many servi, what freemen, how many tenants in soccage, what quantity of wood, how much meadow and pasture, what mills and fish-ponds, how much added or taken away, what the gross value in King Edward's time, what the present value, and how much each freeman, or sock-man had or has. All to be triply estimated; first, as the estate was held in the time of the Conqueror; then as it was bestowed by King William; and thirdly, as its value stood at the formation of the survey. The jury were also to state whether any advance could be made on the value.

Kelsham thinks it probable that the King's Commissioners might find it impossible to take any exact survey of the three most northern counties, as they had suffered so much from the Conqueror's revenge. William of Malmsbury states, so complete was the devastation, that this district, above 60 miles in length, which had been full of towns and cultivated fields, remained barren and desolate even to his time.*

Salt-works occur frequently in the valuation, but they appear to have been only ponds or pans for obtaining salt by evaporation. The rock salt of Cheshire was discovered accidentally in 1670.

Iron and lead works are mentioned, the former frequently; blomes, masses, and plumbæ of iron, are often noticed as rents. Tin is not mentioned, but this may probably be accounted for by the fact that the conquest of Cornwall was completed by Athelstan in 935; was ravaged by the Danes in 997, and again by Godwin and Edwin the sons of Harold in 1068. These misfortunes will sufficiently account for the neglect of the useful arts in that province, at the time of the survey.

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* A. D. 1126.

The lead-works mentioned in the survey are all upon the king's demesne in Derbyshire.

The area of a hide of land is not precisely determined. Polydore Vergil conceived it was derived from the hide of a beast, like the extent of Carthage "quantum taurino possint circundare tergo." But Bishop Kennet says, and with more probability, that it was derived from the Saxon hẏd, a house or habitation, from hẏdan to cover. The word was sometimes used to signify a house, and we still preserve the term "hut" for a cottage. And what Bede calls Familias, (a circuit of land sufficient for the maintenance of a family,) his Saxon translator, king Alfred, calls hẏdelander. The extent was afterwards described to be as much as was sufficient for the cultivation of one plough;* and hence the term Ploughland. The "Dialogus de Scaccario" makes the area one hundred acres. The Malmsbury MSS. cited by Spelman, computes it at 96 acres. The highest authorities however disagree, by much greater quantities than these; and this may have arisen partly from the fact, that then, as now, the area of the acre itself differed in different places, which may not have been sufficiently attended to by men who were not surveyors.

Some authors make the extent as much as six-score acres, others again as little as 20 acres; hence it would appear that the area was not any definite quantity; yet from some entries it appears to have been measured with great precision, even within one foot!

Hutchins in his dissertation on Doomsday says "the hide was the measure of land in the Confessor's reign; the carucate that to which it was reduced by the Conqueror's new standard." Both terms are nevertheless used after the Conquest, and even in the same document; thus among the possessions of Goisfridus de Werce, in Leicestershire, are stated, " VII hides and 1 carucate of land, and one team of oxen."

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* "Hida Anglice vocatur terra unius aratri culturæ sufficiens."

The Carucate, which is considered to be synonymous with Ploughland, was, " as much arable as could be tilled in a year with one plough and the beasts belonging thereto; having pasture and houses appertaining to it, for the occupiers, and the cattle." It was of Norman introduction, and in Doomsday usually follows the measure of the hide; thus every place is said to have paid Geld for so many hides T. R. E.*, and then follows its measure of so many carucates. The area of the carucate, like that of the hide, appears to have varied.

The Solin appears to have been a measure of land known only in the county of Kent; it contained about 200 acres, but the authorities do not agree as to the exact extent.

The Jugum is another unascertained Kentish measure.

The Virgate would appear to have been an aliquot part of a hide, but what part is not determined.

The Bovata was originally as much as an ox-team could plough in a year. 8 bovates are usually said to have made a carucate; but the number of acres in a bovate varies between 8 and 24 in different records.

The term Ferding is used in a few entries in Doomsday, and appears to have been synonymous with Ferling, which is evidently derived from ꝼeoꝛðlınᵹ, or fourth part.

The Acra or acre from the Anglo-Saxon Æceꝛ a field, and that from the Roman Ager, is evidently a very ancient measure. It would appear to have always contained 160 perches; but the perch was of various dimensions.

The length of the Perca, Pertica, or Perch, varied as much in ancient as in modern times, but the present statute perch was established by the " Statutum de admensuratione terrarum," by which it is enacted that 5½ lengths of the Ulna Regis, or yard of 3 feet, were to make one perch.

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* Tempore Regis Edwardi.

The Leuca and Quarentena of Doomsday were generally applied to Woodland. The Leuca, Leuga or Leuua consisted of 480 perches, or 12 quarentenas. Ingulphus speaks of Leuca as of a mile; but Bloomfield renders leuua a league; meaning an extent of about two miles. The mile formerly appears to have been rather a traditionary than an ascertained measure; it was nearly a mile and a half of the present standard.

The Quarentena, called Quadragenaria in the Exeter Doomsday, was the Fortylong or Furlong, from the French quarante, forty, a measure of forty perches.

The several subdivisions of Shires, as Tredings or Ridings, Lests, Lasts or Lathes, Rapes, Wapentakes, Hundreds, Half-hundreds, and Half-lests, are all noticed in Doomsday, but do not appear to be very well defined.

Great light is thrown upon the history of the kingdom by this work, for exclusive of what is stated of the legal privileges, services and transgressions (forisfacturæ) in the different towns and burghs, we find the laws and customs of some of the shires occasionally entered separately; in many instances after the enumeration of the principal burgh customs in each have been given. For instance among other customs of the borough of Lewes it is stated that whosoever sold a horse, in the town, paid 1*d* to the bailiff, and the purchaser paid another. For an ox a halfpenny, and for a *man* 4*d*. The mulct for bloodshed was 7*s* 4*d*. For Adultery or Rape 8*s* 4*d* by each of the parties, and, as in Kent, for Adultery the man became the property of the king, and the woman of the Archbishop. It is not stated to what purposes these forfeitures were applied.

Mills and Woods have been especially noticed.

By the statute of the 9th Ed. II. called " Articuli Cleri" it was determined that prohibition should not lie upon demand of Tithe for a *new* mill; all mills having been exempt from tithe previous to that time. Now a mill mentioned in Doomsday must be presumed to have existed previously to that time; and consequently its being found therein exempts such mill from the payment of Tithe to this day.

Woods appear to have been estimated in many instances by the number of pigs capable of being fed by their acorns and beech-masts. The Saxons (says Nichols) even estimated the value of the tree by the number of hogs that could lie under it.

By this survey the crown acquired an exact knowledge of its possessions, and of the strength and wealth of the country. It afforded moreover a register of appeal for those whose titles to their property might be disputed.

Blackstone has observed that, from the great slaughter of English nobility at the battle of Hastings, such numerous forfeitures had accrued, that William was enabled to reward his Norman followers with very extensive possessions.

It appears from Robertus Montensis that a survey similar to Doomsday was made of Normandy by Henry II.

The Doomsday book, in one instance, designates itself "Liber de Wintonia," which would appear to imply that the original document was lodged at Winchester; yet Ingulphus, who lived in the time of the Conqueror, appears to have obtained extracts from it in London; and the history of the foundation of Burton Abbey refers, more than once, to lands described in *Libro de Domusdie* at Winchester and Westminster. Hence it would appear there was a copy, but of this we have no direct evidence.

In the year 1696 it was deposited in the Chapter House of Westminster, where it still remains.

The Doomsday book, was, among other Records, published in the year 1783; having been ten years in passing through the press.

The Dean and Chapter of Exeter possess a document called the "Exon Doomsday," presenting a description of the Western parts of the Kingdom. It is supposed, so far as it extends, to contain an exact transcript of the original returns from which the great Doomsday book was compiled.

In the Exon book the live stock, as oxen, sheep, goats, horses, and pigs, are invariably enumerated.

The "Inquisitio Eliensis," is a book of the same kind as the Exon book. It relates to the property of the monastery of Ely; and is now preserved among the Cotton MSS. in the British Museum.

The Winton Doomsday, taken in the time of Henry I. is a survey of the King's demesne in Winchester.

The "Boldon Book," is a Survey of the Palatinate of Durham; it was made in the year 1183. It appears to have taken its title from the village and parish of Boldon, near Sunderland. The original book has been lost sight of, but three copies of it still remain; one with the Bishop, and one with the Dean and Chapter of Durham; the other at Oxford.

These books are all written in black letter, in the Latin language, and full of the usual abbreviations; so that literal extracts would be unintelligible to the general reader.

OLD SURVEYS OF IRELAND.

In Ireland there have been several Surveys, so called. It is surmised, from some passages in Giraldus Cambrensus, that a sort of a Doomsday Book was composed, under the influence and orders of Henry Fitz-Empress, upon his arrival in that kingdom, as far, at least, as the English acquisitions extended. The formation of such a document would appear very probable, but if it ever had existence it is now utterly lost.

In the reign of Charles I. a survey of forfeited lands was made by direction of the unfortunate Lord Strafford, and hence called the "Strafford Survey;" but in what manner it was executed is not known, as it, together with many other very important documents, was destroyed in 1711, by the burning of a house in Essex Street, Dublin, where the Surveyor-General's office was then kept.

The most extensive and valuable survey of Ireland, previous to our own times, was that which is called the Down Survey, undertaken by Sir William Petty, Physician-General, under a commission dated 11th December, 1654. It was executed in consideration of a payment of 20*s*. by the day, and of 1*d*. an acre for the grantees obtaining possession of the lands. This survey does not include the whole of the island, but solely the lands, extensive though they were, forfeited by the treasons, real or imputed, of their owners. It was made with the chain and circumferentor,* and laid down with great accuracy, considering the period at which it was executed. The existing parts of the Down Survey, and, it is to be presumed such parts as were destroyed by the fire in 1711, consisted of two sets of maps; one set containing a map of each barony, into which the several counties are divided; and the other a separate map of each parish. The baronial maps are in general on a scale of 160 plantation perches to an inch; but in some the scale is so small as 320 perches to an inch. The parochial maps, existing, are in general on a scale of 80, but some of them of 40 perches, to an inch. The references contain, besides the quantities of land, the names of the former proprietors, together with those of the persons to whom the new grant was made. The persons employed to make this survey were, in many cases, foot-soldiers, incapable of laying it down on paper, which was therefore done by other persons, in the capital. The maps of the Down Survey are admitted as evidence in courts of law, having been lodged, pursuant to act of 14 and 15 Charles II., in his Majesty's surveyor general of lands, and auditor general's offices, "there to remain of record."

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* Vide Report, 1824, from the Select Committee of the House of Commons, on the Survey and Valuation of Ireland, p. 78.

Copies of the baronial maps were taken for Sir William Petty's own use, and were by him forwarded to England; but were intercepted on their passage by a French privateer, carried into France, and deposited in the King's library; where they were seen by Colonel Blaquiere, who, when Irish secretary to Lord Harcourt in 1774, interested government to apply for their restoration.

The French government acceded formally to the request, but evaded the performance of their promise, by asserting that the maps had been mislaid; the truth was, however, that they were put into the hands of Delisle, a French Geographer, for the purpose of constructing a map of Ireland; but the maps were found not to be sufficiently extensive for that purpose.

In perusing a catalogue of the French king's library, some years afterwards, General Vallancey found them there enumerated. Through his representations another application was made to the French, in consequence of which, copies were obtained of those maps; but these copies are not admitted as evidence.

The original Down Survey,* lodged in the surveyor-general's office, consisted of 31 folio volumes. The scale and magnetic meridian are laid down on each map.

Of the authenticated maps, at present, the number is about 1430; of these about 260 are of the baronies, and the remaining 1170 of the parishes. About 67 of the baronial maps are burned, with more or less injury; 130 are fully preserved, and two are stated to be lost: of the parochial maps, about 391 have suffered from fire, and about 780 have altogether escaped. These maps are deposited among the records in the Record Tower, and are evidence in courts of law, as between the Crown and those deriving under it; as also between two persons both deriving title under the Crown.

The last official survey of an extensive nature made in Ireland was that of the forfeited lands, in the reign of William III; it is preserved in the Vice-treasurer's office, and comprises about 2,000,000 of acres. The whole surface of Ireland comprises about 20,000,000 statute acres.

ORDNANCE SURVEY OF IRELAND.

This survey, which has been mentioned in other parts of this volume, is now (1842) nearly completed, but no official account of the work has yet been published. The English triangulation was extended across St. George's Channel in the year 1824, so that both islands may be said to be included in one survey; but this identity does not extend beyond the primary triangulation. The measurement of Ireland has been executed altogether by military men. The survey of Ireland is essentially a territorial one; that of England a topographical one. On the English plans the boundaries of counties only are shewn; on the Irish ones are laid down those of counties, baronies, parishes, and townlands.

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* The title is supposed to have arisen from the fact of the survey being laid down on paper.

The ascertaining of these boundaries was, in some cases, a matter of considerable difficulty, but to meet it, a number of civilians were appointed, under Richard Griffith, esquire. These civilians, many of whom had previously been surveyors, preceded the military, ascertaining and marking the boundaries to be surveyed. The following is the manner in which the work was proceeded with.

Mr. Griffith, chief boundary surveyor, appointed four district surveyors, who were each allotted a district, extending over a considerable area. Under these were appointed, also by Mr. Griffith, assistant boundary surveyors. The number of assistants varied, according to circumstances.

The first thing to be done, on entering a district, was to obtain as many existing plans, whether public or private, as possible. In some cases the county maps were of great assistance, but in many instances next to none. In some cases the parishes had been surveyed for the purposes of taxation, or for the proprietors.

Then, an assistant having been appointed to a parish, he collected all the plans, if any, relating to it, and proceeded to the locality. When there he applied to the clergyman, resident proprietor, or other influential person, and ascertained who was likely to have the best personal knowledge of the lands; this generally turned out to be a tithe proctor, or a baronial constable, or one of their assistants; sometimes a herd, or under steward. The boundary surveyor then, accompanied by this meresman, and a labourer or two, with spades, and provided with a field-book, and tracings of such plans as he had been able to procure, commenced his perambulation, marking and sketching the boundaries as he went. Distinct marks were cut with the spade at every material angle; having arms pointing in the direction of the boundaries. The boundary surveyor made a careful hand sketch as he proceeded, shewing the position of all his marks; and, generally, pacing the distances. The meresman was instructed to observe the merings and the marks, so as to be able to point them out afterwards to the ordnance surveyors.*

Having perambulated all the boundaries in the parish, the surveyor proceeded to make his sketch-map. This was accomplished by first connecting such plans as he had been able to obtain, and so fixing as many points as possible, from which to get in his sketch. If no private plans had been found, then the county map was resorted to; and the roads and rivers constituted the frame work. But few cases occurred in which some assistance could not be obtained towards keeping the points in something like their relative positions. Much depended on the tact of the boundary surveyor. All existing plans having been constructed in plantation measure, the Sketch-maps were made to a scale of forty plantation perches to an inch; that being the best approximation to the Ordnance scale of six inches to the statute mile. The Sketch-maps were, as soon as completed, sent in to the district boundary surveyor, who, having duly examined them, forwarded them as required to the Captain of Engineers, of the district.

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* In bogs and mountains, by very far the greater number of boundaries had never been settled. The adjusting of these constituted a heavy part of the duty of the boundary surveyor. In some instances he was assisted—or perhaps perplexed—by natural, or traditionary boundaries; but in general it was necessary to make a survey, and to allot the unsettled land according to the best of his judgment.

The Ordnance department of the survey is conducted under the direction of Colonel Colby, under whom were four Captains of Districts. Under each captain were a number of Lieutenants, generally of Royal Engineers, with some few Lieutenants of Artillery. Under each lieutenant were a number of the corps of Sappers and Miners; together with a few civil assistants.

The primary triangulation was performed under the personal superintendence of Colonel Colby. The secondary triangulation by the captains, personally. And the tertiary or parochial triangulation, by the respective lieutenants, personally.

The angles having been observed, the sides of the several triangles were calculated by trigonometry, and laid down on paper. The Colonel furnishing the captains with the lengths of such lines, in the primary series, as they required. And the captains furnishing the lieutenants with lines of the secondary series. The lieutenants then laid down the triangulation of each parish, in itself.

A parish having been allotted to a lieutenant, he received the Sketch-map, &c., and proceeded to the ground; and having found the certified meresman, perambulated the boundaries with him. The trigonometrical points were decided upon, and marked, generally with young fir trees, as soon as possible. The angles were then taken, and the triangulation laid down. The calculations were made by young civilians, engaged for this exclusive purpose, at a very low rate of remuneration. These lads acquired great facility, from practice. It was not unusual for one of them to work out as many as forty angles in an hour.

The trig. stations having been decided upon, the parish was apportioned to the Sappers of the party; who with the meresman perambulated their respective portions, and proceeded with the chain-survey. If the measured length differed from the calculated length of a line by ten links in a mile, it was remeasured. Instances have occurred in difficult places of lines being remeasured several times.

The field-books of the Sappers were then handed to the draughtsmen, who were for the most part Civil Assistants; the lieutenants superintending and aiding in the drawing.

The quantities are ascertained by reducing the irregular figures to triangles, on the paper; and checking by the contents of the large triangles, found trigonometrically.

A parish having been completed, the plans are forwarded to the General Survey Office, Phœnix Park, Dublin, where after due examination they are engraved, to the same scale, viz. six inches to the statute mile.

It is scarcely necessary to say that these engraved plans may be purchased by any person, at a very moderate rate.

These plans are furnished to a corps of Valuators, appointed by, and under the direction of Mr. Griffith. The boundaries furnish so many fixed points, that it is not very difficult to subdivide a parish into any required number of holdings, nor to lay down the "Quality lines" with sufficient precision.

All local taxation will hereafter be founded on this survey and valuation.

CADASTRE OF FRANCE.

Ancient surveys of foreign countries are not of the same interest to us as those of our own; but the survey or Cadastre of France, now in progress, is a very important work; and is the more interesting from its having been the moving cause, and indeed the foundation of the Ordnance surveys of these kingdoms.

The French Cadastre, which is composed of the acreage partition, and of the correct delineation of each proprietorship, has been organized by the establishment of a school of Geography, and the creation of a central and general agency of contributions. This agency, known under the name of the Bureau, for the direction of the direct contributors, is placed under the immediate authority of the Minister of Finance, and consists of a director, an inspector, and a comptroller, and about sixty members; who are charged with the surveillance of the department employed in taking the acreage, and in uniting uniformity of principles, by communication with the best informed; and by a coincidence of ideas, which has in particular contributed to the perfect success of these operations. The valuation of the lands, is confined to the municipal council, which is composed of the Mayor and his assistants, and of twelve persons, selected from amongst thirty of the most intelligent persons residing within the commune; and the distribution of the lands, according to their qualities, by three resident proprietors of each commune, who are selected by the municipal council, and who are assisted by the comptroller of the direct contributors. In this same way errors are rectified which may have occurred in measuring the boundaries of proprietorships, on the complaint to this effect of any proprietor, being addressed to the prefêt. *La contribution fonciere* is calculated on an uniform principle, and is of a certain variable number of additional centimes, above the contributions exacted by government, to cover the expenses of the communes: the proportions of tax to territorial income is generally about one-ninth to one-tenth. In the valuation, the number of classes of land have been limited to five, from the nature of the culture. There are consequently five classes of meadow and pasture, and five of arable land.

The maps are lithographed, and are beautifully executed, to a scale of four centimetres to one hundred metres;* which is equal to about 25⅓ inches to an English mile.

The Cadastre of France was first commenced in 1796, under an order of the Directory; but Bonaparte not approving of the mode in which it was conducted, abandoned that work, and commenced the present survey in 1807. The scale being so large, enables the surveyor to exhibit with accuracy, the smallest divisions of property. France contains about 128,000,000 of English acres, the estimated expense of the survey is £4,680,000, or rather more than 8¾*d* per acre.

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* Minutes of evidence before Select Committee of the House of Commons, on the Survey and Valuation of Ireland, p. 17, &c.

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THE END.

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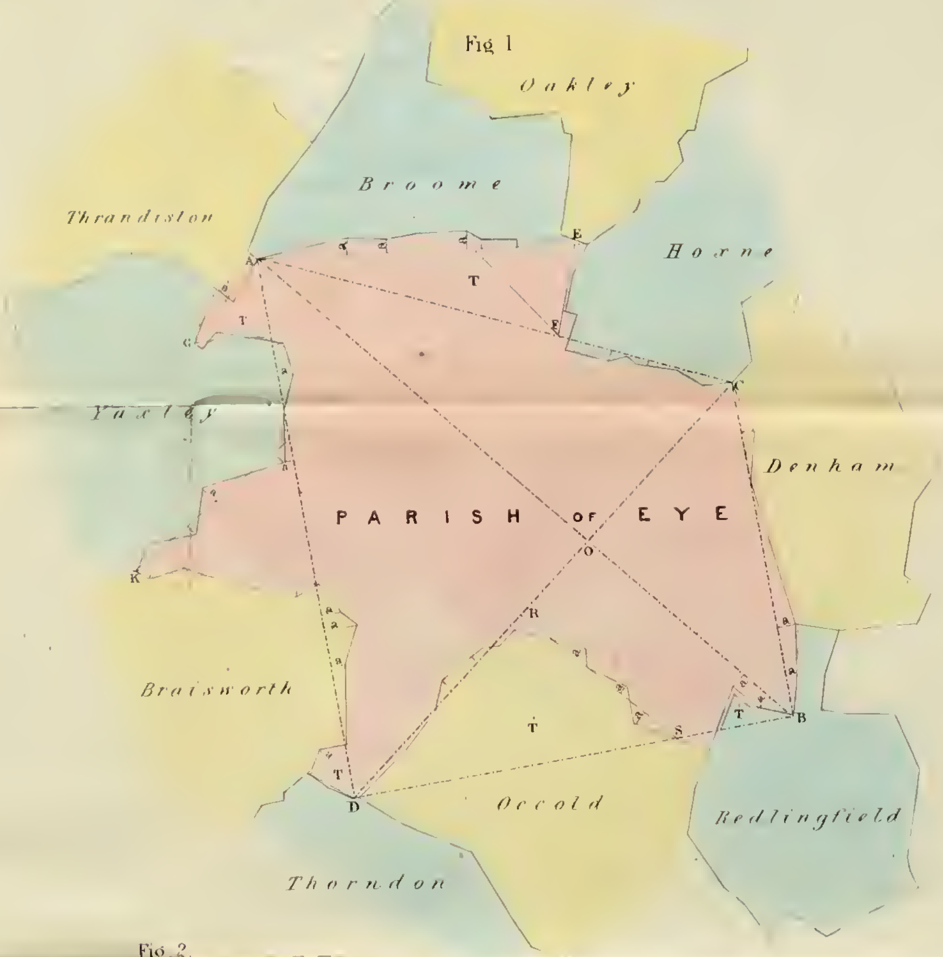
EXAMPLES OF PRIMARY TRIANGULATION.

Plate I

Fig 1



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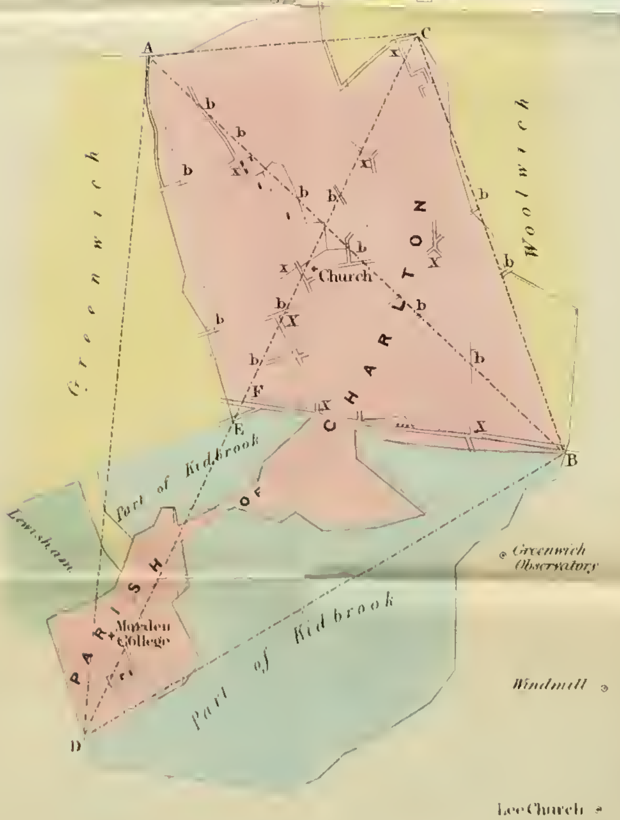
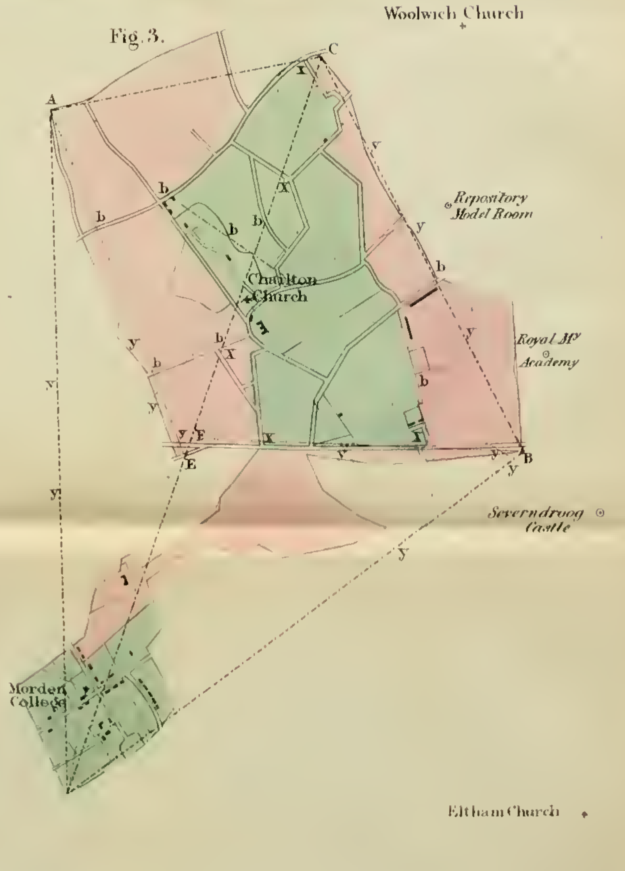


Fig. 3.



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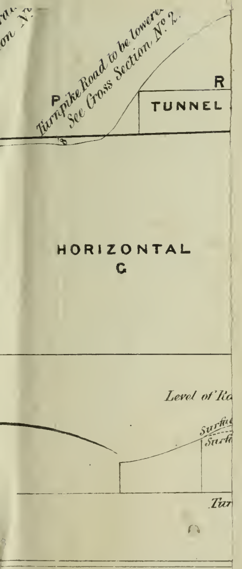


on No.
Turnpike Road to be lowered
See Cross Section No. 2.
P
R
TUNNEL
HORIZONTAL
G
Level of Ro
Surfa
Surfa
Tur

PARLIAMENTARY PLAN and SECTION.

Horizontal 0 ¼ ½ ¾ 1 Mile 4 Inches

Vertical 0 10 20 30 40 50 100 Feet 1 Inch

Scales for Plan and Sections

FIG. 1.

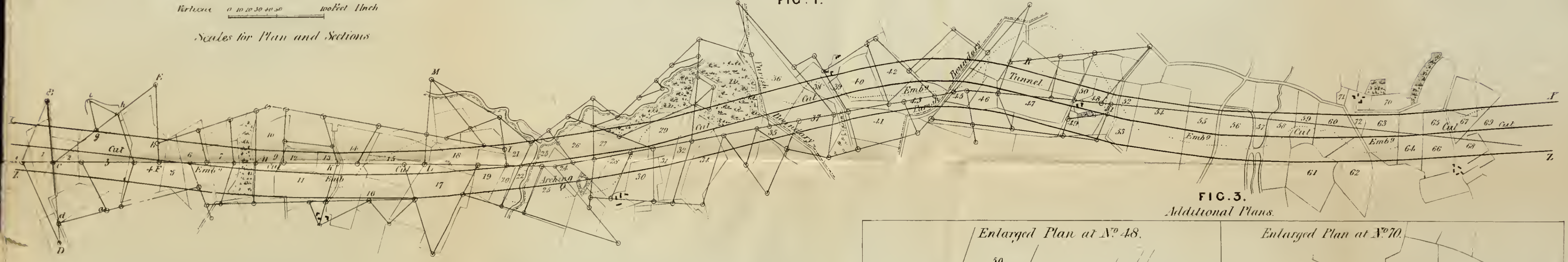


FIG. 3.

Additional Plans.

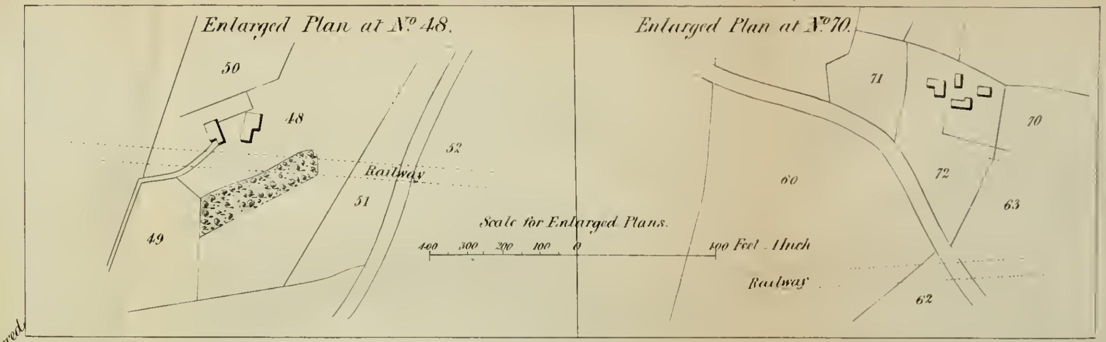


FIG. 5.

PARISH MAP.

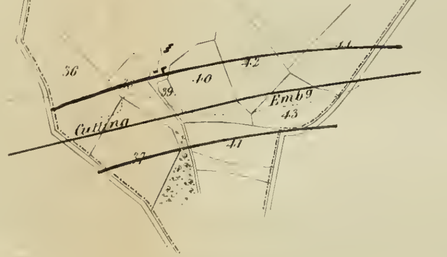
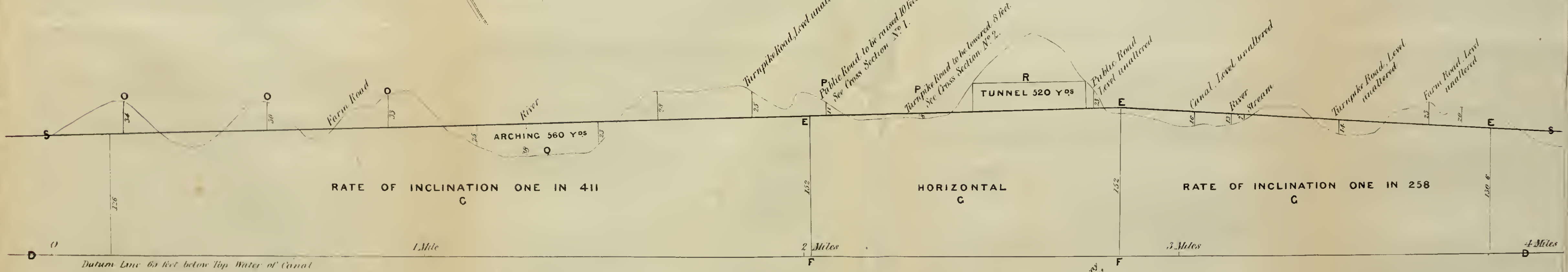
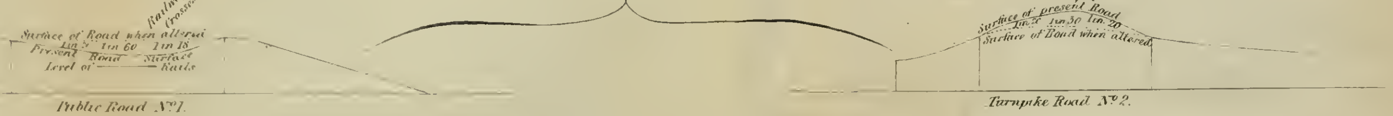


FIG. 2.



Turnpike Road, Level unaltered
Public Road to be raised 10 feet See Cross Section N.º 1.
Turnpike Road to be lowered 8 feet See Cross Section N.º 2.
Public Road Level unaltered
Canal, Level unaltered
River
Stream
Turnpike Road, Level unaltered
Farm Road Level unaltered
Farm Road
River
O
O
O
P
P
R
TUNNEL 520 Yᴰˢ
ARCHING 560 Yᴰˢ
Q
S
S
E
E
E
34
30
33
25
28
25
17
23
16
12
7
14
33
20
RATE OF INCLINATION ONE IN 411
C
HORIZONTAL
C
RATE OF INCLINATION ONE IN 258
C
126
152
152
130 6
0
1 Mile
2 Miles
3 Miles
4 Miles
D
D
F
F
Datum Line 65 feet below Top Water of Canal

FIG. 4.



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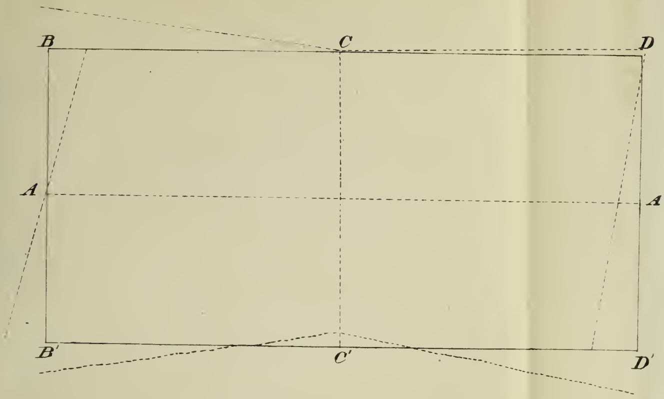
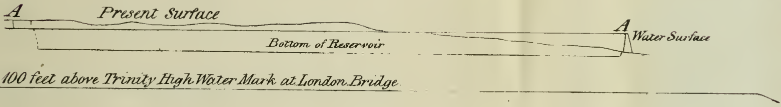
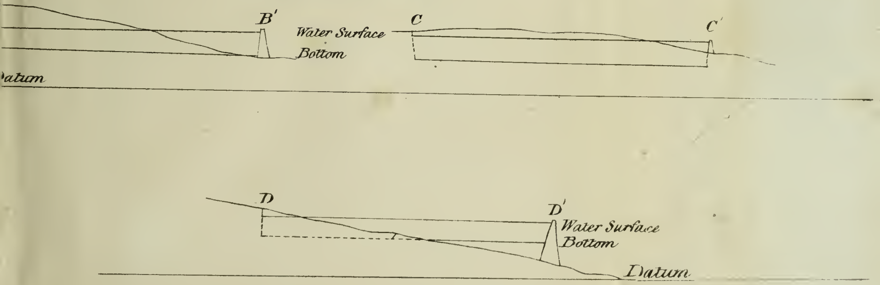
PLAN OF RESERVOIR.

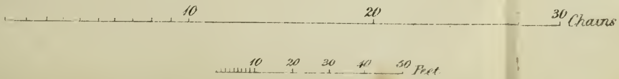


SECTION.


A
Present Surface
A
Water Surface
Bottom of Reservoir
Datum 100 feet above Trinity High Water Mark at London Bridge

CROSS SECTIONS.





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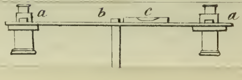
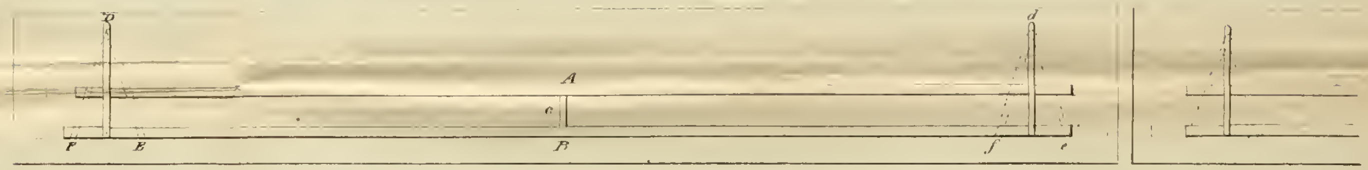

a
b
c
a

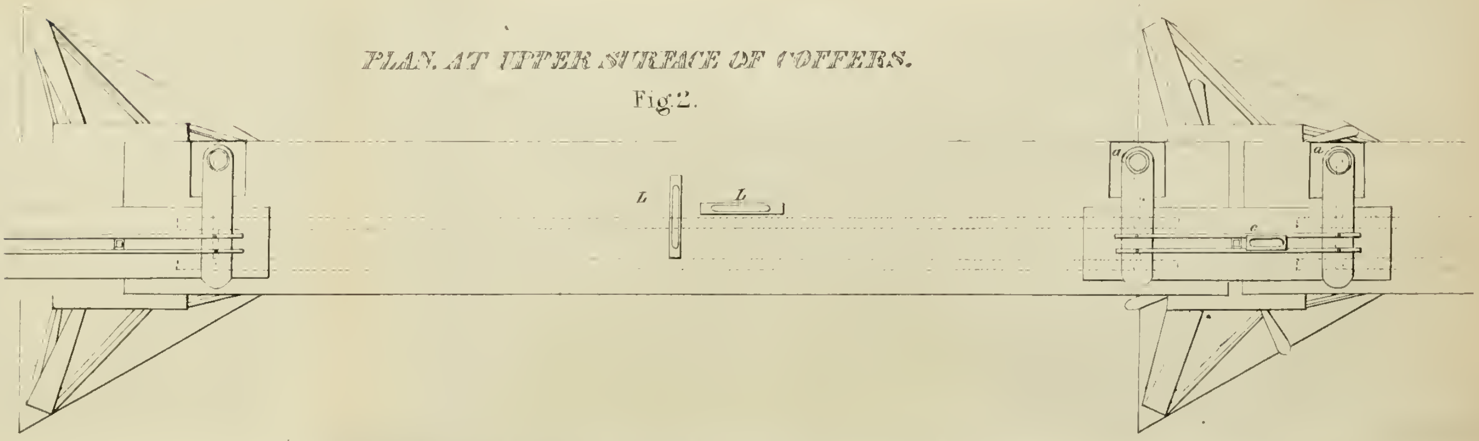
APPARATUS, USED IN MEASURING THE BASE LINE ON THE TRIGONOMETRICAL SURVEY OF IRELAND.

Fig 1.



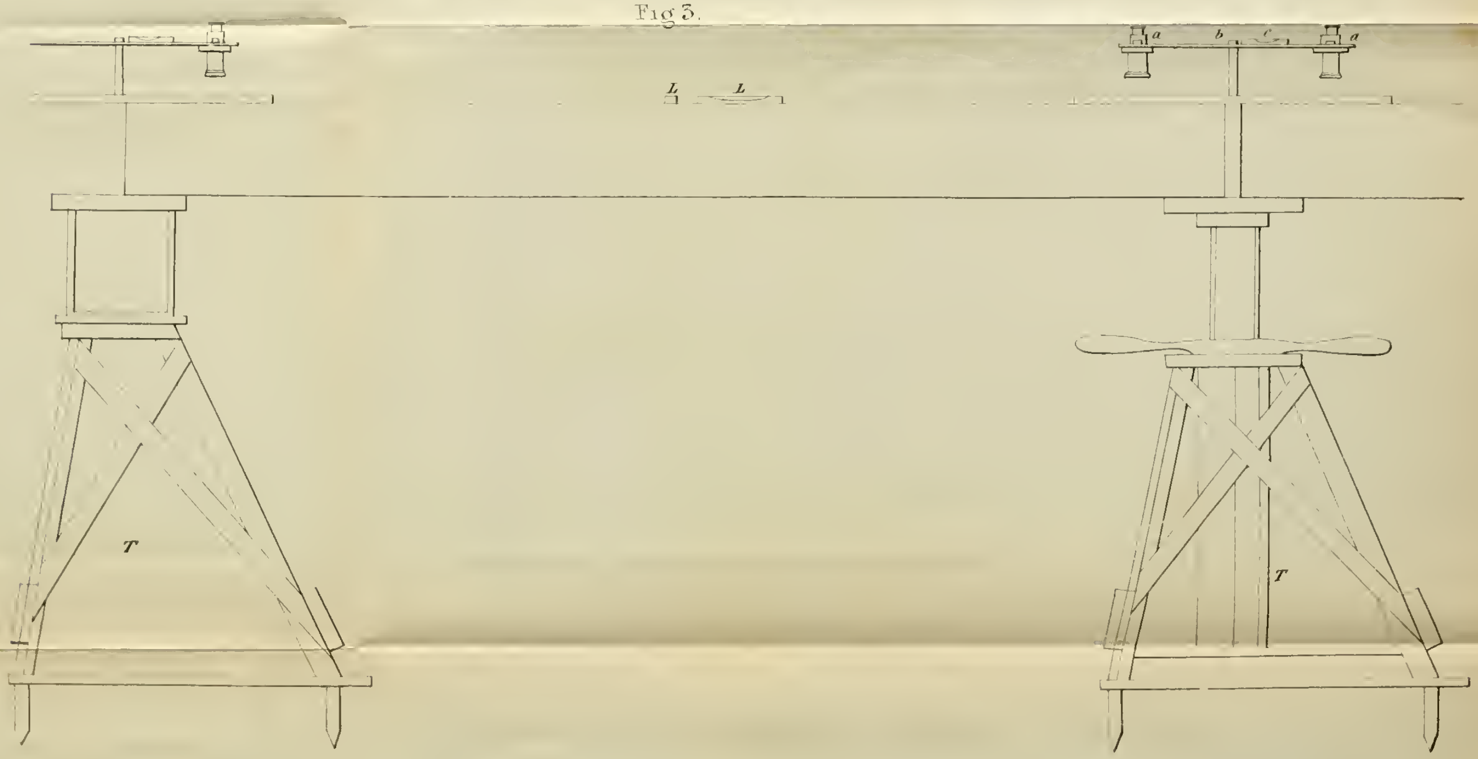
PLAN. AT UPPER SURFACE OF COFFERS.

Fig. 2.



ELEVATION, SHEWING MICROSCOPES, TRESTLES, &C.

Fig 3.



Drawn by Walter Bryden

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PLATE VIII.

SURVEY OF THE RIVER SHANNON.

KILBRIDE DEER PARK

PLAN AND SECTIONS OF SHOAL

Showing the Proposed Improvements
1838.

WILLIAM T. MULVANY, Civil Engineer.

Scale for the Plan and Lengths of the Longitudinal Section

50 0 50 100 150 200 250 300 350 400 450 500 Feet

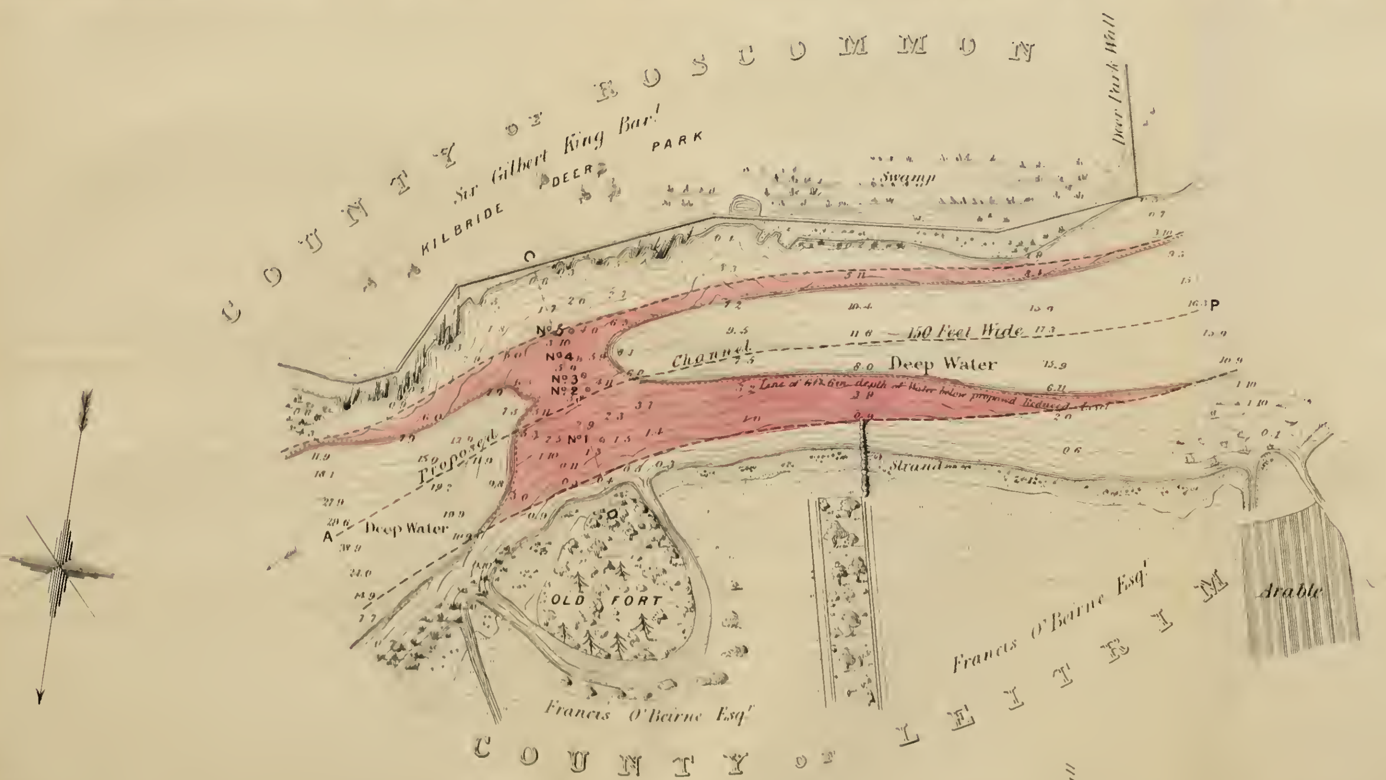
Scale for the Heights of the Sections and for the Lengths of the Transverse Section.

10 5 0 10 20 30 40 50 60 70 80 90 100 Feet

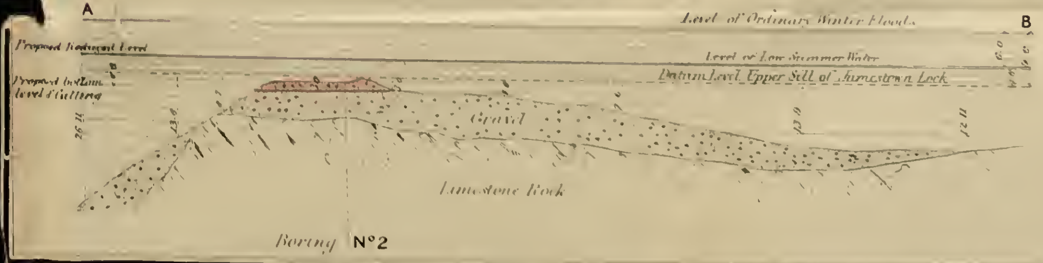
NOTE.

The Proposed Improvements are shown in Red, the Soundings are marked in Feet and Inches, and represent the Depths of Water below the Proposed Reduced Level (3ft 6in on the present Upper Sill of Jamestown Lock) which is also the Low Summer Water of this place.

The Sites of the several Borings which have been made in this Shoal are represented on the Plan thus, N° 6 ○ on the Sections thus Gravel Boring N°5



LONGITUDINAL SECTION
IN THE LINE A. B.


A
B
Level of Ordinary Winter Flood
Proposed Reduced Level
Level or Low Summer Water
Datum Level Upper Sill of Jamestown Lock
Proposed bottom level of Cutting
Gravel
Limestone Rock
Boring N°2

TRANSVERSE SECTION
IN THE LINE C. D

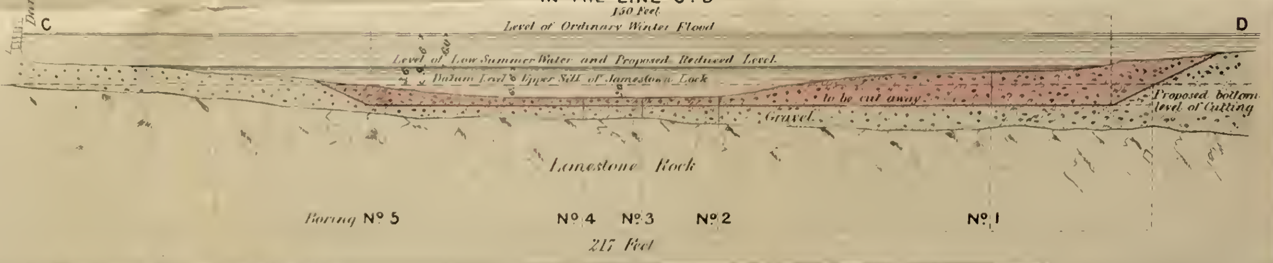

150 Feet
Level of Ordinary Winter Flood
C
D
Deer Park Wall
Level of Low Summer Water and Proposed Reduced Level
Datum Level Upper Sill of Jamestown Lock
to be cut away
Proposed bottom level of Cutting
Gravel
Limestone Rock
Boring N° 5
N° 4
N° 3
N° 2
N° 1
217 Feet

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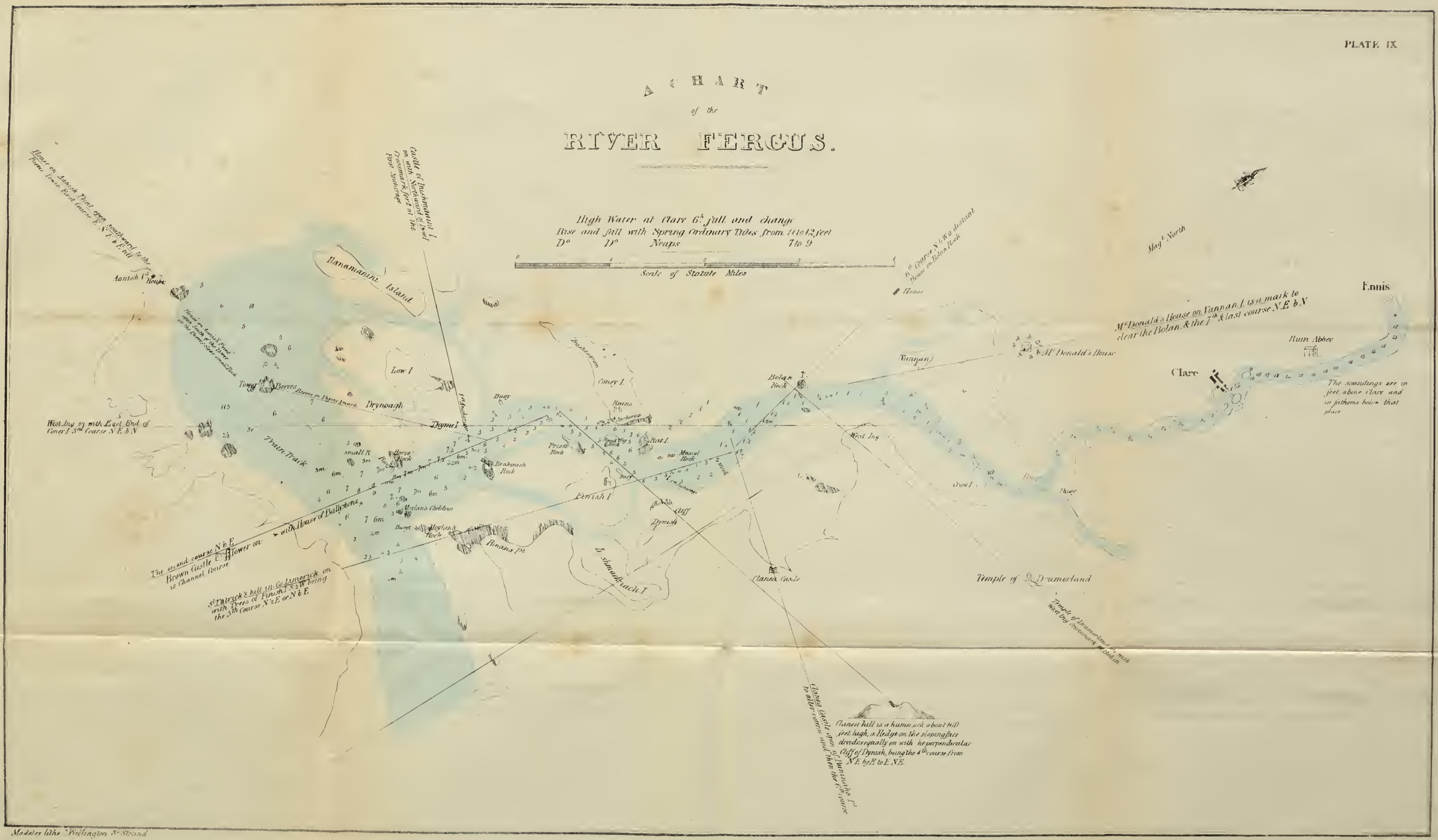
PLATE IX



A CHART of the RIVER FERGUS.

High Water at Clare 6h full and change
Rise and fall with Spring Ordinary Tides from 11to12 feet
D° D° Neaps 7 to 9

0
1
2
3
Scale of Statute Miles
Magt North
House on Aanish Point open southward to the Beeves Tower First course E N E b E till
Aanish Pt House
House on Aanish Point open South of the Tower on the Beeves clear small Rock
Castle of Inishmaunn I on with Northward of Bowl Crossmark fort at the First Anchorage
Ilanamanini Island
Low I
Tower
Beeves
Beeves on Dyme I mark
Drynoagh
Dyme I
1st Anchorage
Buoy
West Ing on with East End of Coney I 3rd Course N E b N
Train Track
small R
Horse Rock
Brakenish Rock
Moylan's Children
Moylan's Rock
Buoy
Rinana Pt
The second course N b E
Brown Castle Tower on with House of Ballystone is Channel Course
St Patrick's hill in Co Limerick on with Trees of Finish I S ½ W being the 5th Course N ½ E or N b E
Inishtagram
Coney I.
Ruins
Anchorage
Priest Rock
Road Way Rock
Rat I.
Mascel Rock
Finish I
Low Anchorage
mud
Cliff
Dynish I
Ishmacknack I
Clarea Castle
Bolan Rock
West Ing
Crow I
Vannan I
6th Course N b W a distant House on Bolan Rock
House
Mc Donald's House
Mc Donald's House on Vannan I. is a mark to clear the Bolan, & the 7th & last course N E b N
Ruin Abbey
Clare
Ennis
Buoy
The soundings are in feet above Clare and in fathoms below that place
Temple of Drumorland
Temple of Drumorland on with West Ing crossmark on Bolan
Clarea Castle open of Bunnatha Pt to alter course and then the 6th course
Claree hill is a hummock about 160 feet high, a Hedge on the sloping face divides equally on with the perpendicular Cliff of Dynish, being the 4th course from N E by E to E N E.

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